

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01T

Fall Term 2004

Problem Set 9: Momentum and Collision Theory : Solutions

Problem 1: Impulse and Momentum

The compressive force per area necessary to break the tibia in the lower leg, is about $F/A = 1.6 \times 10^8 \text{ N} \cdot \text{m}^2$. The smallest cross sectional area of the tibia, about 3.2 cm^2 , is slightly above the ankle. Suppose a person of mass $m = 6.0 \times 10^1 \text{ kg}$ jumps to the ground from a height $h_0 = 2.0 \text{ m}$ and absorbs the shock of hitting the ground by bending the knees. Assume that there is constant deceleration during the collision. During the collision, the person lowers his center of mass by an amount $\Delta d = 1 \text{ cm}$.

- a) What is the collision time Δt_{col} ?
- b) Find the average force of the ground on the person during the collision.
- c) What is the average impulse of the ground on the person?
- d) Will the person break his ankle? How much would you need to lower your center of mass so you do not break your ankle?

a) The speed of the person before touching the ground is $\sqrt{2gh_0}$

If a is the magnitude of the deceleration we must have:

$$a\Delta t_{coll} = \sqrt{2gh_0} \text{ and } \frac{1}{2}a(\Delta t_{coll})^2 = (\Delta t_{coll})\sqrt{\frac{gh_0}{2}} = \Delta d \text{ from which}$$

$$\Delta t_{coll} = \Delta d \sqrt{\frac{2}{gh_0}} = 3.2 \cdot 10^{-3} \text{ s}$$

$$a = \frac{gh_0}{d}$$

b) The average force of the ground on the person is:

$$|F| = m(g + a) = mg \frac{d+h_0}{d} = 118 \cdot 10^3 \text{ N}$$

c) The average impulse of the ground on the person is:

$$|I| = |F|\Delta t_{coll} = 379 \text{ N s}$$

d) the force required to break the tibia is:

$$F_B = (1.6 \cdot 10^8)(3.2 \cdot 10^{-4}) \text{ N} = 51.2 \cdot 10^3 \text{ N}$$

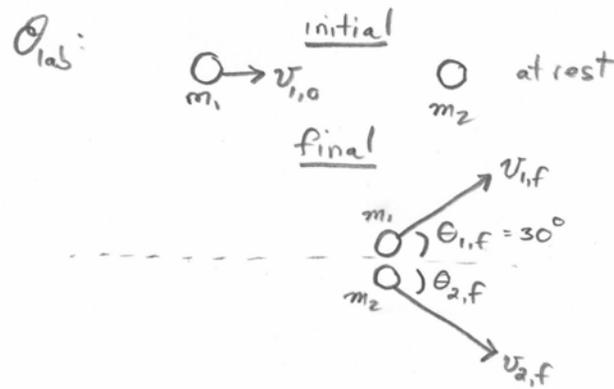
so that the tibia will break (Ouch!)

In order not to suffer the person should lower its center of mass at least of:

$$\Delta d = \frac{mgh_0}{F_b - mg} = 2.3 \text{ cm}$$

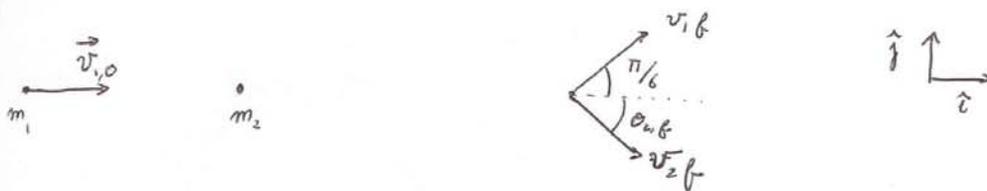
Problem 2: Two Dimensional Elastic Collision

An object with mass m_1 is initially moving with a velocity $v_{1,0} = 3.0\text{m/s}$ and collides elastically with another object of equal mass $m_2 = m_1$ that is initially at rest. After the collision m_1 moves with an unknown speed $v_{1,f}$ at an angle $\theta_{1,f} = 30^\circ$ with respect to its initial direction of motion. After the collision, m_2 moves with an unknown speed $v_{2,f}$, at an unknown angle $\theta_{2,f}$ (see sketch). Find the final velocities of each of the masses and the angle $\theta_{2,f}$.



Naturally we can solve the problem also entirely in the lab frame.

However this requires a good quantity of trigonometric alchemy:



$$\vec{P}_i = m_1 \vec{v}_{1,0} = m_1 |\vec{v}_{1,0}| \hat{i}$$

$$\vec{P}_f = m_1 \vec{v}_{1,f} + m_2 \vec{v}_{2,f} = m_1 (v_{1,f} \sin \theta_{1,f} + v_{2,f} \sin \theta_{2,f}) \hat{j} + m_1 (v_{1,f} \cos \theta_{1,f} + v_{2,f} \cos \theta_{2,f}) \hat{i}$$

$$\vec{P}_i = \vec{P}_f \Rightarrow \frac{1}{2} v_{1,0} = v_{2,f} \sin \theta_{2,f} \quad \text{from the } \hat{j} \text{ direction.}$$

$$v_{1,0} = \frac{\sqrt{3}}{2} v_{1,f} + v_{2,f} \cos \theta_{2,f} \quad \text{from the } \hat{i} \text{ direction.}$$

$$\begin{aligned} \Rightarrow v_{1,0} &= \sqrt{3} v_{2,f} \sin \theta_{2,f} + v_{2,f} \cos \theta_{2,f} = 2 v_{2,f} \left(\frac{\sqrt{3}}{2} \sin \theta_{2,f} + \frac{1}{2} \cos \theta_{2,f} \right) = \\ &= 2 v_{2,f} \sin \left(\theta_{2,f} + \frac{\pi}{6} \right) \quad v_{2,f} = \frac{v_{1,0}}{2 \sin \left(\theta_{2,f} + \frac{\pi}{6} \right)} \quad v_{1,f} = \frac{v_{1,0} \sin \theta_{2,f}}{\sin \left(\theta_{2,f} + \frac{\pi}{6} \right)} \end{aligned}$$

Then we can use conservation of energy to write

$$v_{1,0}^2 = \frac{v_{1,0}^2}{4 \sin^2 \left(\theta_{2,f} + \frac{\pi}{6} \right)} + \frac{v_{1,0}^2 \sin^2 \theta_{2,f}}{\sin^2 \left(\theta_{2,f} + \frac{\pi}{6} \right)} \Rightarrow 4 \sin^2 \left(\theta_{2,f} + \frac{\pi}{6} \right) = 1 + 4 \sin^2 \theta_{2,f}$$

$$\begin{aligned} 1 - \cos \left(2\theta_{2,f} + \frac{\pi}{3} \right) &= \frac{1}{2} + 1 - \cos(2\theta_{2,f}) \quad \cos(2\theta_{2,f}) - \cos \left(2\theta_{2,f} + \frac{\pi}{3} \right) = \frac{1}{2} \\ 2 \sin \left(2\theta_{2,f} + \frac{\pi}{6} \right) \sin \left(\frac{\pi}{6} \right) &= \frac{1}{2} \quad \sin \left(2\theta_{2,f} + \frac{\pi}{6} \right) = \frac{1}{2} \quad \theta_{2,f} = \frac{\pi}{3} \text{ or } \theta_{2,f} = 0 \end{aligned}$$

$\theta_{1,f} = 0$ represents the trivial solution (the two balls collide head on and the first stops and in fact $v_{1,f} = 0$ in this case)

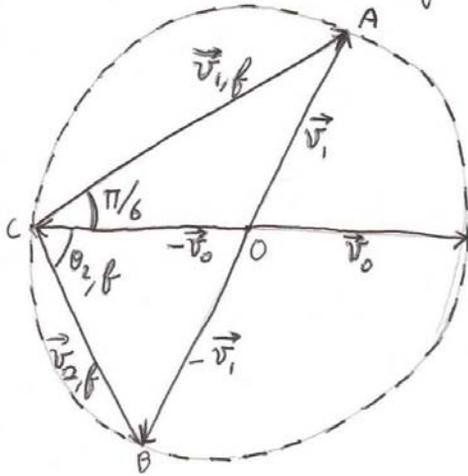
$\theta_{2,f} = \frac{\pi}{3}$ is more interesting, In this case.

$$v_{1,f} = \frac{v_{1,0}}{2} = 1.5 \text{ ms}^{-1} \quad | \quad v_{2,f} = v_{1,0} \frac{\sqrt{3}}{2} = 2.6 \text{ ms}^{-1} \quad \text{as before.}$$

Alternative solution in center of mass frame:

To solve the problem we will define $\vec{v}_0 = \frac{1}{2} \vec{v}_{1,0}$ the speed of the first ball in the c.m. frame, Then the second ball will have speed $-\vec{v}_0$. Setting the speed of the first ball after the collision in the c.m. frame as \vec{v}_1 we will have from momentum conservation that the second ball bounces with speed $-\vec{v}_1$. Then from energy conservation

$|\vec{v}_0| = v_0 = v_1 = |\vec{v}_1|$. The final speeds of the balls in the Lab frame (see figure) are: $\vec{v}_{1,f} = \vec{v}_1 - (-\vec{v}_0)$ $\vec{v}_{2,f} = -\vec{v}_1 - (-\vec{v}_0)$



Now $\frac{\pi}{6} + \theta_{2,f} = \frac{\pi}{2}$ because $\hat{A}BC = \frac{\pi}{2}$ as A, B, C lie on a circle of center O and $\hat{A}OB = \pi$

$$\theta_{2,f} = \frac{\pi}{3}$$

$$|\vec{v}_{1,f}| = 2 v_0 \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} v_{1,0} = 2.6 \text{ ms}^{-1}$$

$$|\vec{v}_{2,f}| = 2 v_0 \cos \frac{\pi}{3} = \frac{1}{2} v_{1,0} = 1.5 \text{ ms}^{-1}$$

Problem 3: Experiment 9: Collisions

Part One Inelastic Collision:

Analysis: Complete the analysis of your data table by following the two steps below, and answer Question 1 below.

You will analyze this collision as seen by two different observers. One observer is at rest with respect to the classroom. This reference frame is called the lab reference frame. The observer is moving at a velocity equal to the center of mass velocity of the two cart system. This frame is called the center of mass reference frame.

Step 1: Draw two momentum diagrams for both carts immediately before and immediately after the collision, one describing the collision in the lab reference frame, and one describing the collision in the center of mass reference frame.

Answer:

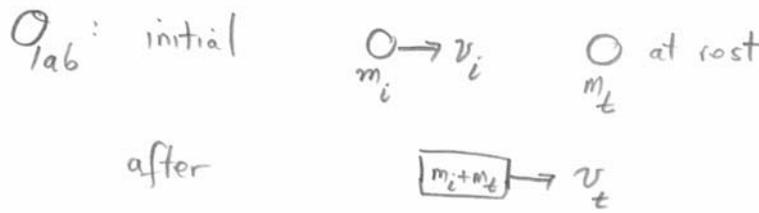


Figure 1: Collision in lab reference frame

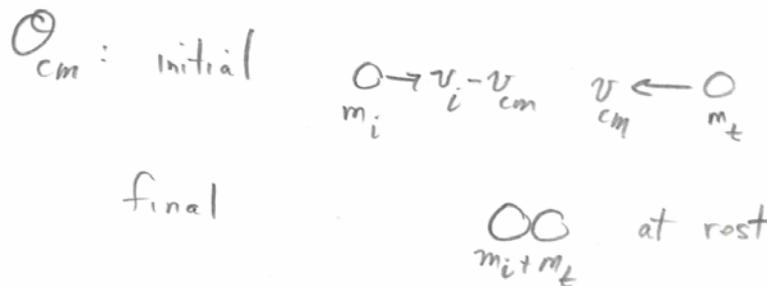


Figure 2: Collision in center of mass reference frame

You made three different trial collisions; the first with equal masses, and the second and third with unequal masses. Use your measured values to complete the following table. Entries to the left of the double vertical bar are before the two carts collide; those to the right are after the two carts collide but before the collision with the force sensor. Hand the table in with your homework solutions.

Step 2:

- a) First calculate the initial kinetic energy K_1 and the final kinetic energy K_2 in the ‘lab reference frame’. The difference between the two, $K_2 - K_1 = W_{nc}$, is the non-conservative work internal work, $W_{nc} = K_f - K_0$, due to the totally inelastic collision. This work is due to forces that irreversibly deform the bodies during the collision.
- b) Then calculate the velocity of the center of mass which is given by

$$\vec{v}_{cm} = \frac{m_A \vec{v}_{A,1}}{m_A + m_b}.$$

Because momentum is constant during the collision, you should expect $\vec{v}_{cm} = \vec{v}_2$. Calculate the total kinetic energy of the two carts in the center of mass reference frame, given by

$$K_{CMCS} = \frac{1}{2} m_A (v_{A,1} - v_{cm})^2 + \frac{1}{2} m_B (v_{cm})^2$$

m_i [kg]	m_t [kg]	v_i [m·s ⁻¹]	v_{cm} [m·s ⁻¹]	K_i [J]	K_{cmcs} [J]	v_f [m·s ⁻¹]	K_f [J]	W_{nc} [J]
0.25	0.25	0.60	0.30	0.0450	0.0225	0.30	0.0225	0.0225
0.25	0.50	0.66	0.22	0.0540	0.037	0.21	0.0165	0.038
0.50	0.25	0.43	0.29	0.0462	0.023	0.27	0.0273	0.019

Question 1: In the center of mass reference frame: the velocity of the incident cart A moves forward with a slower velocity, $\vec{v}_{A,1} - \vec{v}_{cm}$, than in the lab frame; the cart B moves backward with a velocity, $-\vec{v}_{cm}$; After the collision, the two carts are stuck together and are at rest! So the final kinetic energy is zero, $K_f' = 0$. Therefore the non-conservative work in the center of mass reference frame is given by

$$W_{nc} = K_f' - K_{CMCS} = -K_{CMCS}.$$

Show that $K_f - K_0 = -K_{cmcs}$. This will require a calculation. Does your experimental data verify this theoretical result?

Solution: For the first two cases the data agrees with the theory. For the third case when the incident mass is twice the target mass, the final velocity is less than the center of mass velocity

indicating that there is some additional loss of energy due to friction. Since the target is more massive, the incoming energy loss due to friction may be greater.

In the center of mass reference frame with

$$v_{cm} = \frac{m_i v_i}{m_i + m_t}, \text{ the incident cart has}$$

$$\text{velocity } v_i - v_{cm} = v_i - \frac{m_i v_i}{m_i + m_t} = \frac{m_t v_i}{m_i + m_t}$$

The kinetic energy in the center of mass frame is then

$$(K_{cm,cs})_0 = \frac{1}{2} m_i (v_i - v_{cm})^2 + \frac{1}{2} m_t v_{cm}^2$$

$$= \frac{1}{2} m_i \left(\frac{m_t v_i}{m_i + m_t} \right)^2 + \frac{1}{2} \left(\frac{m_i m_t v_i}{m_i + m_t} \right)^2$$

$$= \frac{1}{2} \frac{m_i m_t (m_i + m_t)}{(m_i + m_t)^2} v_i^2 = \frac{1}{2} \frac{m_i m_t}{m_i + m_t} v_i^2$$

The quantity $\frac{m_i m_t}{m_i + m_t}$ is often called the

reduced mass. Since the two carts are at rest in the center of mass reference frame after the collision, $(K_{cm,cs})_f = 0$

$$W_{nc} = \Delta E = \Delta K_{cm} = -(K_{cm,cs})_0$$

In the lab (ground) reference frame
 momentum conservation implies

$$m_i v_i = (m_i + m_t) v_f \Rightarrow v_f = \frac{m_i v_i}{m_i + m_t} = v_{cm}$$

The final carts are moving with the
 center of mass velocity. The initial kinetic
 energy is

$$K_0 = \frac{1}{2} m_i v_i^2$$

The final kinetic energy is

$$\begin{aligned} K_f &= \frac{1}{2} (m_i + m_t) v_{cm}^2 = \frac{1}{2} (m_i + m_t) \frac{m_i^2 v_i^2}{m_i + m_t} \\ &= \frac{1}{2} \frac{m_i^2 v_i^2}{m_i + m_t} \end{aligned}$$

So the change in kinetic energy is

$$\begin{aligned} W_{nc} = \Delta K &= K_f - K_0 = \frac{1}{2} \frac{m_i^2 v_i^2}{m_i + m_t} - \frac{1}{2} m_i v_i^2 \\ &= \frac{1}{2} m_i v_i^2 \left(\frac{m_i}{m_i + m_t} - 1 \right) = -\frac{1}{2} \frac{m_i m_t v_i^2}{m_i + m_t} \\ &= -(K_{cm,cs})_0 \end{aligned}$$

Since the carts are constrained to move along the track, there is no rotation in this collision. The displacement of an individual particle during the collision in the lab reference frame can be considered to be a vector sum of the displacement relative to the center of mass, and a displacement due to the center of mass motion,

$$d\vec{r}_{i,lab} = d\vec{r}_{i,cm} + d\vec{r}_{cm}$$

The total internal non-conservative force acting on the particle i due to the interaction with particle j does a work equal to

$$\vec{F}_{i,j} \cdot d\vec{r}_{i,lab} = \vec{F}_{i,j} \cdot d\vec{r}_{i,cm} + \vec{F}_{i,j} \cdot d\vec{r}_{cm}.$$

The total internal non-conservative force acting on the particle j due to the interaction with particle i does a work equal to

$$\vec{F}_{j,i} \cdot d\vec{r}_{j,lab} = \vec{F}_{j,i} \cdot d\vec{r}_{j,cm} + \vec{F}_{j,i} \cdot d\vec{r}_{cm}.$$

When we add these terms together, the second terms cancel,

$$\vec{F}_{i,j} \cdot d\vec{r}_{cm} + \vec{F}_{j,i} \cdot d\vec{r}_{cm} = 0,$$

due to the fact that the forces are equal and opposite by Newton's Third Law,

$$\vec{F}_{i,j} - \vec{F}_{j,i}.$$

Therefore the contribution to the total work is

$$\vec{F}_{i,j} \cdot d\vec{r}_{i,cm} + \vec{F}_{j,i} \cdot d\vec{r}_{j,cm} \neq 0.$$

This is not zero because the individual particles do not necessarily displace the same amount. This is the result of the deformation of the body,

$$d\vec{r}_{i,cm} \neq d\vec{r}_{j,cm}$$

So when we sum over all particle pairs this gives the total internal work. This only depends on the motion relative to the center of mass, so in either reference frame, the internal work is the same, hence the change in kinetic energy should also be the same.

Part Two Elastic Collision:

Analysis: Complete the analysis of your data table by following the two steps below, and answer Question 2-5 below.

Once again you made three trials. Since the motion sensor can only calculate the initial and final velocities of the incident cart, you needed to measure the impulse of the second cart on the force sensor to determine the velocity of the target cart.

Step 1: Draw a momentum diagram for the collision immediately before the cart B hits the force sensor and after cart B rebounds from the force sensor.

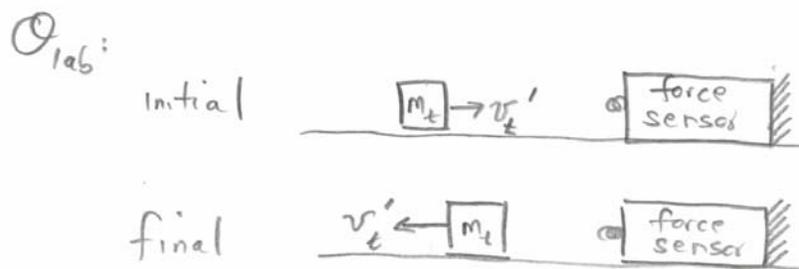


Figure 3: Elastic collision of Cart B with force sensor

Step 2: If we assume that cart B collides elastically with the force sensor, the momentum of the cart B changes by $\Delta\vec{p}_B = -2m_B\vec{v}_{B,2}$. This change in momentum is the impulse that the force sensor exerts on the target cart. The target cart therefore exerts an equal but opposite impulse on the force sensor with magnitude $J = 2m_B v_{B,2}$. So measuring the impulse allows you to calculate the velocity of the target cart, $v_{B,2} = J/2m_B$. Use your measured values to complete the following table. Velocity entries to the left of the double vertical bar are before the two carts collide; velocity entries to the right are after they collide but before the collision with the force sensor. Hand the table in with your homework solutions.

m_i [kg]	m_t [kg]	v_i [$m \cdot s^{-1}$]	v_{cm} [$m \cdot s^{-1}$]	v_i' [$m \cdot s^{-1}$]	J [$kg \cdot m \cdot s^{-1}$]	v_t' [$m \cdot s^{-1}$]
0.25	0.25	0.53	0.265	0.00	0.27	0.54
0.25	0.75	0.50	0.125	-0.16	0.31	0.20
0.75	0.25	0.38	0.095	0.17	0.26	0.54

Question 2: Assuming that the collision is elastic, based on the principles of conservation of momentum and conservation of energy, calculate the theoretical final velocities for cart B and cart A for the three trials. Show your work. (These calculations are involved for the second and third trials.) Briefly describe whether or not your measured values agree with your calculated values from part d. Are the collisions elastic?

$$\text{of (3)} \Rightarrow v_i' = 3v_t' - v_i$$

$$\text{of (4)} \Rightarrow v_i^2 = (3v_t' - v_i)^2 + 3v_t'^2$$

$$v_i^2 = 9v_t'^2 - 6v_t'v_i + v_i^2 + 3v_t'^2$$

$$\text{or } 0 = 12v_t'^2 - 6v_t'v_i$$

$$\text{Solution: } \underbrace{v_t' = \frac{1}{2}v_i}_{\text{final state}}, \quad \underbrace{v_t' = 0}_{\text{initial state}}$$

$$v_i' = 3v_t' - v_i = \frac{3}{2}v_i - v_i = \frac{1}{2}v_i$$

They both rebound with the same speed but opposite directions.

iii) when $3m_t = m_i$

$$\text{cons. of mom} \Rightarrow 3v_i = v_t' - 3v_i' \quad (5)$$

$$\Rightarrow v_t' = 3(v_i + v_i')$$

$$\text{cons. of energy} \Rightarrow 3v_i^2 = 3v_i'^2 + v_t'^2 \quad (6)$$

$$\text{which becomes } 3v_i^2 = 3v_i'^2 + 9(v_i^2 + 2v_iv_i' + v_i'^2)$$

or

$$0 = 12v_i'^2 + 18v_iv_i' + 6v_i^2$$

$$\text{this factors } 0 = (6)(2v_i' + v_i)(v_i' + v_i)$$

Solutions: $v_i' = -\frac{1}{2}v_i$ or $v_i' = -v_i$
final state initial state

initial cart moves in + direction with speed $\frac{1}{2}v_i$

Then $v_t' = 3(v_i + v_i') = 3(v_i - \frac{1}{2}v_i) = \frac{3}{2}v_i$

For the first case, the final velocity of the incident cart was zero, and the target cart was traveling at nearly the initial velocity of the incident cart. The collision was elastic. For the second case, the final velocity of the both the incident cart and the target cart were smaller than the predicted $0.5v_i$ indicating that the collision was not elastic. For the third case, the target cart was 5% slower than the predicted value, while the incident cart was 11% slower, again indicating that the collision was not quite elastic.

Part Three: Modeling the Elastic Collision

Question 3: The repulsive force between the two carts is modeled by the force law

$$F = Ae^{-x/l}$$

where x is the separation between the carts and l is a characteristic length associated with the collision. If the carts start very far apart (an infinite distance), the work required to bring them to a separation distance d is

$$W = \int_{\infty}^d F dx = \int_{\infty}^d Ae^{-x/l} dx.$$

Calculate this integral. The result is the equivalent of the potential energy stored in a spring $(1/2)kx^2$ that satisfies a linear restoring force law.

Solution: The change in potential energy in bringing the carts from an infinite distance apart to a distance d apart is the negative of the work done by the repulsive magnetic force

$$\Delta U = -\int_{\infty}^d F dx = -\int_{\infty}^d Ae^{-x/l} dx = -\frac{Ae^{-x/l}}{(-1/l)} \Big|_{x=\infty}^{x=d} = lAe^{-d/l}.$$

Question 4: Suppose there are two of these carts on a track, one cart with mass m and speed $v_{A,1}$ runs into the other, which has equal mass and is initially at rest. Find the minimum separation d between the carts during the collision. You may assume that A is sufficiently large that so that $d > l$. Evaluate your result numerically for $A = 2.0 \times 10^1 \text{ N}$, $l = 5.0 \times 10^{-3} \text{ m}$, $m = 2.5 \times 10^{-1} \text{ kg}$, and $v_{A,1} = 5.0 \times 10^{-1} \text{ m} \cdot \text{s}^{-1}$. Hint: This problem is most easily done in the center of mass reference frame.

Solution: In the center of mass reference frame, the energy is constant so

$$\Delta K + \Delta U = 0.$$

We already so that the initial kinetic energy in the center of mass frame is given by

$$K_{cm,i} = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} v_i^2 = \frac{1}{4} m v_{A,1}^2.$$

The carts are at rest in the center of mass frame when they reach closet approach.

$$K_{cm,f} = 0.$$

So

$$\Delta K = -\frac{1}{4} m v_{A,1}^2,$$

Thus the constant energy condition becomes

$$-\frac{1}{4} m v_{A,1}^2 + l A e^{-d/l} = 0.$$

Thus

$$e^{-d/l} = \frac{1}{4lA} m v_{A,1}^2.$$

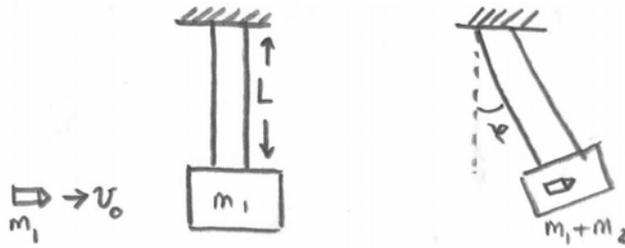
Taking natural ln of both sides and solving for d yields

$$\begin{aligned} d &= -l \ln\left(\frac{1}{4lA} m v_{A,1}^2\right) \\ &= (-5.0 \times 10^{-3} \text{ m}) \ln\left(\frac{(2.5 \times 10^{-1} \text{ kg})(5.0 \times 10^{-1} \text{ m} \cdot \text{s}^{-1})^2}{(4)(5.0 \times 10^{-3} \text{ m})(2.0 \times 10^1 \text{ N})}\right) \\ &= 9.3 \times 10^{-3} \text{ m} \end{aligned}$$

Problem 4: (Ballistic Pendulum)

A simple way to measure the speed of a bullet is with a ballistic pendulum. This consists of a wooden block of mass m_1 into which a bullet of mass m_2 is shot. The block is suspended from two cables, each of length L . The impact of the bullet causes the block and embedded bullet to swing through a maximum angle φ . The initial speed of the bullet is v_0 .

- a) How fast is the block and embedded bullet moving immediately after the bullet comes to rest in the block? Assume that the time the bullet takes to come to rest is negligibly small. Express your answer in terms of the quantities m_1 , m_2 , L , g , v_0 , and φ . You may not need all these quantities.
- b) Find a relation for the initial speed of the bullet as a function of m_1 , m_2 , L , g , and φ .



- a) Using conservation of momentum we have for the speed of the system just after the collision v :

$$v = m_2 \frac{v_0}{m_1 + m_2}$$

- b) Using conservation of energy we have

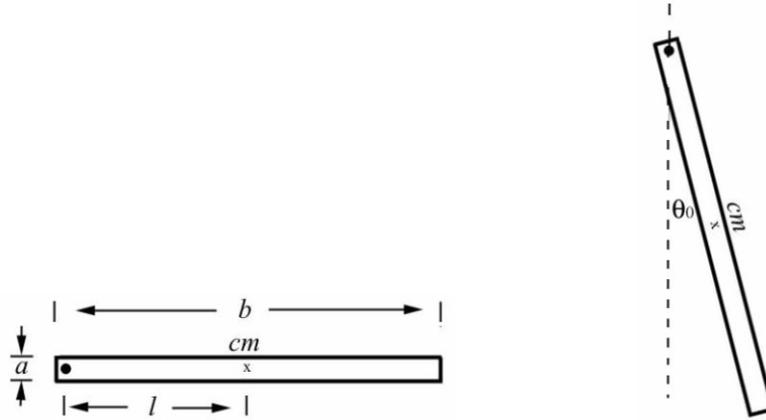
$$(m_1 + m_2)gL(1 - \cos(\theta)) = 2(m_1 + m_2)gL \sin^2\left(\frac{\theta}{2}\right) = \frac{1}{2}(m_1 + m_2)v^2$$

from which:

$$v_0 = (m_1 + m_2) \frac{v}{m_2} = 2 \sin\left(\frac{\theta}{2}\right) \sqrt{gL} \frac{m_1 + m_2}{m_2}$$

Problem 5: Experiment 8 Physical Pendulum Pre-experiment Question

Consider a uniform ruler shown in the figures below. The ruler has mass m_1 , width a and length b . The ruler is pivoted about a point at one end and allowed to oscillate. The pivot point is a distance l from the center of mass.



- a) If the ruler is given a small initial displacement by an angle of 5 degrees to one side, explain why it will undergo simple harmonic motion.
- b) If the initial displacement is increased by an additional twenty degrees will the period increase, decrease, or stay the same?

a) The equation of motion for the angle θ is:

$$I \frac{d^2\theta}{dt^2} = -m_1lg \sin(\theta)$$

where I is the momentum of inertia of the ruler around the suspension point and the right hand side is the torque of the gravitational force around it.

For small θ the equation reduces to:

$$I \frac{d^2\theta}{dt^2} = -m_1lg\theta$$

which is exactly of the form required for harmonic motion with $\omega = \sqrt{\frac{m_1g}{I}}$.

b) If the initial displacement is not small the absolute value of the torque $|m_1lg \sin(\theta)| < |m_1lg\theta|$ so that the period will increase.