

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01T

Fall Term 2004

Problem Set 9: Momentum and Collision Theory

Available on-line October 29; Due: November 9 at 4:00 p.m.

Please write your name, subject, lecture section, table, and the name of the recitation instructor on the top right corner of the first page of your homework solutions. Please place your solutions in your lecture section table box.

Oct 29 No Class

Problem Set 8: Due Tues Nov 2 at 4:00 pm.

Nov 1

Hour One: Conservation Laws: Momentum and Collision Theory.

Reading: YF 8.3-8.6

Hour Two: Problem Solving Session 14: Conservation of Momentum

Reading: YF 8.3-8.6

Nov 3

Hour One: Experiment 7: Collisions

Reading: Experiment 7

Hour Two: Rigid Bodies Kinematics

Reading: YF 9.1-9.6

Nov 5

Hour One: Problem Solving Session 15: Moment of Inertia and Rotational Energy

Problem Set 9: Due Tues Nov 9 at 4:00 pm.

Nov 8

Hour One: Torque, Rotational Dynamics.

Hour Two: Problem Solving Session 16: Rotational Dynamics, Simple Harmonic Motion:
Physical Pendulum

Nov 10

Hour One: Experiment 8: Physical Pendulum

Hour Two: Conservation Laws: Angular Momentum

Nov 12

Hour One: Problem Solving Session 17: Angular Momentum

Problem Set 10: Due Tues Nov 16 at 4:00 pm.

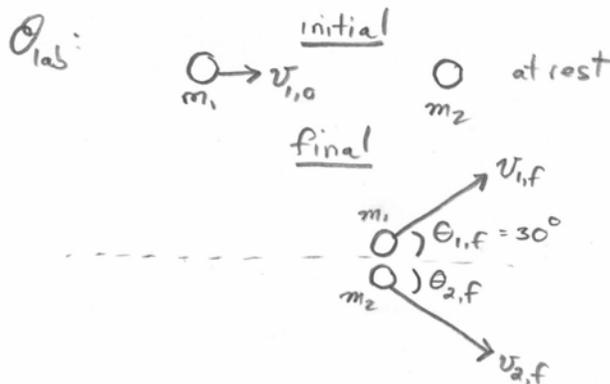
Problem 1: *Impulse and Momentum*

The compressive force per area necessary to break the tibia in the lower leg, is about $F/A = 1.6 \times 10^8 \text{ N} \cdot \text{m}^2$. The smallest cross sectional area of the tibia, about 3.2 cm^2 , is slightly above the ankle. Suppose a person of mass $m = 6.0 \times 10^1 \text{ kg}$ jumps to the ground from a height $h_0 = 2.0 \text{ m}$ and absorbs the shock of hitting the ground by bending the knees. Assume that there is constant deceleration during the collision. During the collision, the person lowers his center of mass by an amount $\Delta d = 1 \text{ cm}$.

- What is the collision time Δt_{col} ?
- Find the average force of the ground on the person during the collision.
- What is the average impulse of the ground on the person?
- Will the person break his ankle? How much would you need to lower your center of mass so you do not break your ankle?

Problem 2: *Two Dimensional Elastic Collision*

An object with mass m_1 is initially moving with a velocity $v_{1,0} = 3.0 \text{ m/s}$ and collides elastically with another object of equal mass $m_2 = m_1$ that is initially at rest. After the collision m_1 moves with an unknown speed $v_{1,f}$ at an angle $\theta_{1,f} = 30^\circ$ with respect to its initial direction of motion. After the collision, m_2 moves with an unknown speed $v_{2,f}$, at an unknown angle $\theta_{2,f}$ (see sketch). Find the final velocities of each of the masses and the angle $\theta_{2,f}$.



Problem 3: Experiment 7: Collisions Analysis

Part One Inelastic Collision:

Analysis: Complete the analysis of your data table by following the two steps below, and answer Question 1 below.

You will analyze this collision as seen by two different observers. One observer is at rest with respect to the classroom. This reference frame is called the lab reference frame. The observer is moving at a velocity equal to the center of mass velocity of the two cart system. This frame is called the center of mass reference frame.

Step 1: Draw two momentum diagrams for both carts immediately before and immediately after the collision, one describing the collision in the lab reference frame, and one describing the collision in the center of mass reference frame.

Step 2: You made three different trial collisions; the first with equal masses, and the second and third with unequal masses. Use your measured values to complete the following table. Entries to the left of the double vertical bar are before the two carts collide; those to the right are after the two carts collide but before the collision with the force sensor. Hand the table in with your homework solutions.

m_A	m_B	v_{A1}	v_{CM}	K_1	K_{CMCS}	v_2	K_2	W_{NC}
0.25 kg	0.25 kg							
0.25 kg	0.50 kg							
0.50 kg	0.25 kg							

a) First calculate the initial kinetic energy K_1 and the final kinetic energy K_2 in the ‘lab reference frame’. The difference between the two, $K_2 - K_1 = W_{nc}$, is the non-conservative work internal work, $W_{nc} = K_f - K_0$, due to the totally inelastic collision. This work is due to forces that irreversibly deform the bodies during the collision.

b) Then calculate the velocity of the center of mass which is given by

$$\vec{v}_{cm} = \frac{m_A \vec{v}_{A,1}}{m_A + m_b}$$

Because momentum is constant during the collision, you should expect $\vec{v}_{cm} = \vec{v}_2$. Calculate the total kinetic energy of the two carts in the center of mass reference frame, given by

$$K_{CMCS} = \frac{1}{2} m_A (v_{A,1} - v_{cm})^2 + \frac{1}{2} m_B (v_{cm})^2$$

Question 1: In the center of mass reference frame: the velocity of the incident cart A moves forward with a slower velocity, $\vec{v}_{A,1} - \vec{v}_{cm}$, than in the lab frame; the cart B moves backward with a velocity, $-\vec{v}_{cm}$; After the collision, the two carts are stuck together and are at rest! So the final kinetic energy is zero, $K_f' = 0$. Therefore the non-conservative work in the center of mass reference frame is given by

$$W_{nc} = K_f' - K_{CMCS} = -K_{CMCS}.$$

Show that $K_f - K_0 = -K_{cmcs}$. This will require a calculation. Does your experimental data verify this theoretical result?

Part Two Elastic Collision:

Analysis: Complete the analysis of your data table by following the two steps below, and answer Question 2 below.

Once again you made three trials. Since the motion sensor can only calculate the initial and final velocities of the incident cart, you needed to measure the impulse of the second cart on the force sensor to determine the velocity of the target cart.

Step 1: Draw a momentum diagram for the collision immediately before the cart B hits the force sensor and after cart B rebounds from the force sensor.

Step 2: If we assume that cart B collides elastically with the force sensor, the momentum of the cart B changes by $\Delta\vec{p}_B = -2m_B\vec{v}_{B,2}$. This change in momentum is the impulse that the force sensor exerts on the target cart. The target cart therefore exerts an equal but opposite impulse on the force sensor with magnitude $J = 2m_B v_{B,2}$. So measuring the impulse allows you to calculate the velocity of the target cart, $v_{B,2} = J/2m_B$. Use your measured values to complete the following table. Velocity entries to the left of the double vertical bar are before the two carts collide; velocity entries to the right are after they collide but before the collision with the force sensor. Hand the table in with your homework solutions.

m_A	m_B	v_{A1}	v_{CM}	K_1	v_{A2}	J	v_{B2}	K_2
0.25 kg	0.25 kg							
0.25 kg	0.75 kg							
0.75 kg	0.25 kg							

Question 2: Assume that the collision is elastic. Use the principles of conservation of momentum and energy to calculate the theoretical final velocities for cart B and cart A for the three trials. Show your work. (These calculations are involved for the second and third trials.) Briefly describe whether or not your measured values agree with your theoretical values. Are the collisions elastic?

Part Three: Modeling the Elastic Collision

Question 3: The repulsive force between the two carts is modeled by the force law

$$F = Ae^{-x/l}$$

where x is the separation between the carts and l is a characteristic length associated with the collision. If the carts start very far apart (an infinite distance), the work required to bring them to a separation distance d is

$$W = \int_{\infty}^d F dx = \int_{\infty}^d Ae^{-x/l} dx.$$

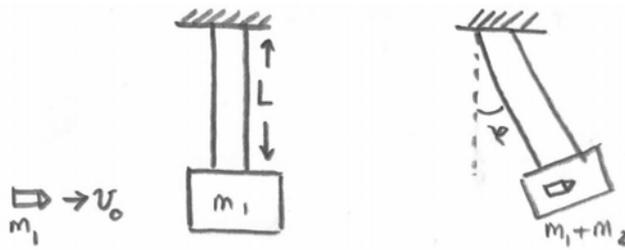
Calculate this integral. The result is the equivalent of the potential energy stored in a spring $(1/2)kx^2$ that satisfies a linear restoring force law.

Question 4: Suppose there are two of these carts on a track, one cart with mass m and speed $v_{A,1}$ runs into the other, which has equal mass and is initially at rest. Find the minimum separation d between the carts during the collision. You may assume that A is sufficiently large that so that $d > l$. Evaluate your result numerically for $A = 2.0 \times 10^1$ N, $l = 5.0 \times 10^{-3}$ m, $m = 2.5 \times 10^{-1}$ kg, and $v_{A,1} = 5.0 \times 10^{-1}$ m · s⁻¹. Hint: This problem is most easily done in the center of mass reference frame.

Problem 4: (Ballistic Pendulum)

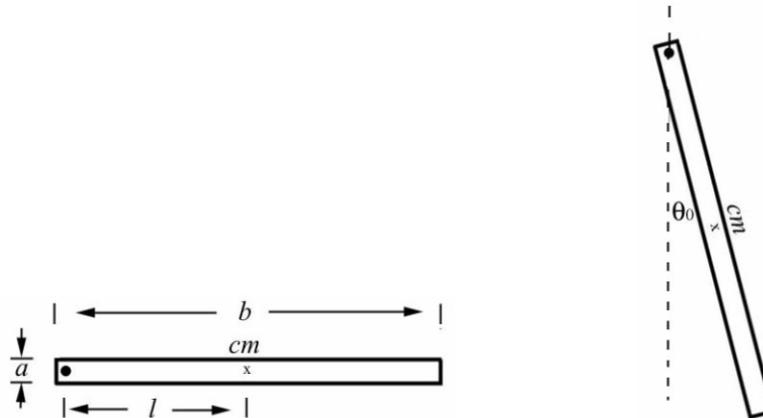
A simple way to measure the speed of a bullet is with a ballistic pendulum. This consists of a wooden block of mass m_1 into which a bullet of mass m_2 is shot. The block is suspended from two cables, each of length L . The impact of the bullet causes the block and embedded bullet to swing through a maximum angle φ . The initial speed of the bullet is v_0 .

- How fast is the block and embedded bullet moving immediately after the bullet comes to rest in the block? Assume that the time the bullet takes to come to rest is negligibly small. Express your answer in terms of the quantities m_1 , m_2 , L , g , v_0 , and φ . You may not need all these quantities.
- Find a relation for the initial speed of the bullet as a function of m_1 , m_2 , L , g , and φ .



Problem 5: Experiment 8 Physical Pendulum Pre-experiment Question

Consider a uniform ruler shown in the figures below. The ruler has mass m_1 , width a and length b . The ruler is pivoted about a point at one end and allowed to oscillate. The pivot point is a distance l from the center of mass.



- If the ruler is given a small initial displacement by an angle of 5 degrees to one side, explain why it will undergo simple harmonic motion.
- If the initial displacement is increased by an additional twenty degrees will the period increase, decrease, or stay the same?