**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**
**Department of Physics**

**Physics 8.01T** **Fall Term 2004**

# Problem Set 8: Center of Mass, Momentum and Impulse: Solutions

## Problem 1: Pre-Experiment 07 Concept Questions

a) Cart A is at rest on a straight horizontal track with negligible friction. An identical cart B, moving to the right, collides with cart A. They stick together. After the collision, which of the following is true?

1. Carts A and B are both at rest.
2. Carts A and B move to the right with a speed greater than cart B's original speed.
3. Carts A and B move to the right with a speed less than cart B's original speed.
4. Cart B stops and cart A moves to the right with speed equal to the original speed of cart B.

Explain your reasoning.

a) The carts stick together, so the collision is inelastic. Using conservation of momentum, we obtain

$$mv_B = 2mv_{AB},$$

where $v_B$ is the velocity of cart B before the collision and $v_{AB}$ is the velocity of the two carts after the collision. Thus $v_{AB} = v_B/2$.

b) Cart A is at rest on a straight horizontal track with negligible friction. An identical cart B, moving to the right, collides elastically with cart A. They do not stick together. After the collision, which of the following is true?

1. Carts A and B are both at rest.
2. Cart B stops and cart A moves to the right with speed equal to the original speed of cart B.
3. Cart A remains at rest and cart B bounces back with speed equal to its original speed.
4. Cart A moves to the right with a speed slightly less than the original speed of cart B and cart B moves to the right with a very small speed.

b) The collision is elastic, so we can use both conservation of momentum and conservation of the kinetic energy. Thus

$$mv_B = mv_A^{(a)} + mv_B^{(a)}$$
$$\frac{mv_B^2}{2} = \frac{m(v_A^{(a)})^2}{2} + \frac{m(v_B^{(a)})^2}{2}$$

where the superscript (a) refers to the velocities after the collision. We square the first equation and subtract the result from the second equation to obtain $v_A^{(a)} v_B^{(a)} = 0$, which means that the velocity of one of the carts after the collision must be zero and the velocity of the other cart must be equal to $v_B$ (momentum is conserved!). Since the carts can not pass through each other, it means that cart B stops and cart A moves to the right with the speed equal to the original speed of cart B.

c) Suppose you drop paperclips into an open cart rolling along a straight horizontal track with negligible friction. As a result of the accumulating paperclips, explain whether the momentum and kinetic energy increase, decrease, or stay the same.

c) The momentum of the cart stays the same provided clips are dropped into the cart, i.e. have zero initial horizontal momentum. Thus

$$m_{cart} v_{initial} = (m_{cart} + m_{clips}) v_{final}$$

and the kinetic energy after the "collision" is

$$\frac{(m_{cart} + m_{clips}) v_{final}^2}{2} = \frac{m_{cart}^2 v_{initial}^2}{2(m_{cart} + m_{clips})} = \frac{m_{cart}}{m_{cart} + m_{clips}} K_{initial} < K_{initial}$$

**Problem 2:** ***Center of Mass***

The mean distance from the earth to the sun is $r_{e,s} = 1.49 \times 10^{11}\, m$. The mass of the earth is $m_e = 5.98 \times 10^{24}\, kg$ and the mass of the sun is $m_s = 1.99 \times 10^{30}\, kg$. The mean radius of the earth is $r_e = 6.37 \times 10^{6}\, m$. The mean radius of the sun is $r_s = 6.96 \times 10^{8}\, m$. Where is the location of the center of mass of the earth-sun system?

The center of mass is located distance

$$d_e = \frac{m_s r_{e,s}}{m_s + m_e} = 1.4899955 \times 10^{11}\, m$$

from the center of the Earth, or equivalently, distance

$$d_s = r_{e,s} - d_e = \frac{m_e r_{e,s}}{m_s + m_e} = 4.48 \times 10^{5}\, m$$

from the center of the Sun.

**Problem 3:** ***Impulse and Momentum***

A ball of mass $m_1 = 8.0 \times 10^{-2}\, kg$, starting from rest, falls a height of $3.0\, m$ and then collides with the ground. The ball bounces up to a height of $2.0\, m$. The collision with the ground takes place over a time, $\Delta t_{col} = 5.0 \times 10^{-3}\, s$.

a) What is the momentum of the ball immediately before the collision?

a) The velocity of the ball before it hits the ground is found using the conservation of energy

$$m v_b^2/2 = mgh$$

Thus the momentum of the ball is $p_b = -m v_b = -m\sqrt{2gh_1} = -0.61\, kg\, m/s$, where $h_1 = 3\, m$. Note that we chose the positive direction to be up.

b) What is the momentum of the ball immediately after the collision?

b) Again, we use the conservation of energy to find $p_a = m v_a = m\sqrt{2gh_2} = 0.5\, kg\, m/s$ with $h_2 = 2\, m$.

c) What is the average force of the ground on the ball?

c) The average force is

$$\bar{F} = \frac{\Delta p}{\Delta t} = m\sqrt{2g}\frac{\sqrt{h_2} + \sqrt{h_1}}{\Delta t} = 223\,N$$

d) What impulse is imparted to the ball?

d) The impulse is equal to $\Delta p = 1.1\,kg\,m/s$.

e) What is the change in the kinetic energy during the collision?

e) The change in the kinetic energy is

$$\Delta K = \frac{mv_a^2}{2} - \frac{mv_b^2}{2} = \frac{p_a^2}{2m} - \frac{p_b^2}{2m} = mgh(h_2 - h_1) = -0.78\,J$$

**Problem 4:**

A block of mass $m_b$ on a horizontal table is connected to one end of a spring with spring constant $k$ . The other end of the spring is attached to a wall. The block is set in motion so that it oscillates about its equilibrium point with amplitude $A_0$ .

a) What is the period of the motion?

a) The period of the motion is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m_b}{k}}$$

A lump of sticky putty of mass $m_p$ is dropped onto the block. The putty sticks without bouncing. The putty hits the block at the instant when the velocity of the block is zero.

b) Find the new period, the new amplitude, and the change in mechanical energy of the system.

b) When the putty hits the block, the velocity of the block is zero. Therefore the string is stretched or compressed to maximum. The putty does not transfer any momentum to the block, thus the amplitude and the mechanical energy stay the same. The new period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m_b + m_p}{k}}$$

The putty is removed and the block is set in motion with the same amplitude $A_0$. This time the putty is dropped and hits the block at the instant the block has its maximum velocity.

c) Find the new period, the new amplitude, and the change in mechanical energy of the system.

c) The period is again given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m_b + m_p}{k}}.$$

We can use the result of Problem 1 c) to find the change of the mechanical energy after the collision

$$\Delta E = \Delta K = K_f - K_i = \frac{m_b}{m_b + m_p}K_i - K_i = -\frac{m_p}{m_p + m_b}K_i = -\frac{m_p}{m_p + m_b}\frac{kA_0^2}{2}.$$

The new amplitude is found using

$$\frac{kA^2}{2} = K_f = \frac{m_b}{m_b + m_p}\frac{kA_0^2}{2}.$$

Thus

$$A = \sqrt{\frac{m_b}{m_b + m_p}}A_0$$