**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**
**Department of Physics**

**Physics 8.01T** **Fall Term 2004**

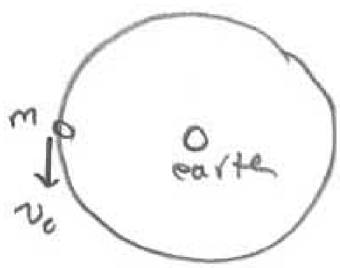
# Problem Set 7: Potential Energy, Change in Mechanical Energy Solutions

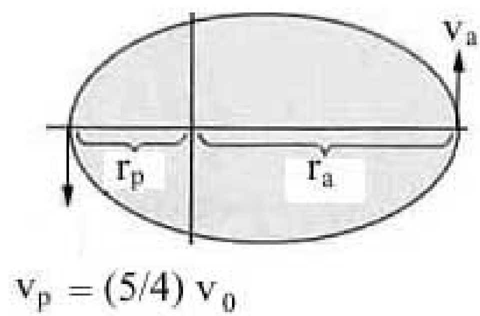
## Problem 1: *Gravitational Potential and Kinetic Energy*

A satellite of mass $m = 5.0 \times 10^{2}\, kg$ is initially in a circular orbit about the earth with a radius $r_0 = 4.1 \times 10^{7}\, m$ and velocity $v_0 = 3.1 \times 10^{3}\, m/s$ around the earth. Assume the earth has mass $m_e = 6.0 \times 10^{24}\, kg$ and radius $r_e = 6.4 \times 10^{6}\, m$. Since $m_e >> m$, you may find it convenient to ignore certain terms. Justify any terms you choose to ignore.

a) What is the magnitude of the gravitational force acting on the satellite? What is the centripetal acceleration of the satellite?

b) What are the kinetic and potential energies of the satellite earth system? State any assumptions that you make. Specify your reference point for zero potential energy. What is the total energy?




As a result of a satellite maneuver, the satellite trajectory is changed to an elliptical orbit. This is accomplished by firing a rocket for a short time interval and increasing the tangential speed of the satellite to $(5/4)v_0$. You may assume that during the firing, the satellite does not noticeably change the distance from the center of the earth.

c) What are the kinetic and potential energies of the earth-satellite at the point of closest approach? Specify your reference point for zero potential energy. What is the total energy?

d) How much energy was necessary to change the orbit of the satellite?

e) What speed would the satellite need to acquire so that it can escape to infinity?

a) The magnitude of the gravitational force is $|\overrightarrow{F}| = \frac{Gm_s m_e}{r_0{}^2} = 117\ N$

The centripetal acceleration is $\overrightarrow{a} = -\frac{v_0^2}{r_0}\hat{r}$ and $\frac{v_0^2}{r_0} = 0.23\ ms^{-2}$

b) We will suppose the earth as fixed and we will choose the 0 point for the potential energy to be $\infty$. Then we have $K = \frac{1}{2}m_s v_0^2 = 24\ 10^8 J$ and $U = -\frac{Gm_s m_e}{r_0} = -2K = -48\ 10^8 J$

c) At the point of closest approach that is just after the rocket goes off we have

$K = \frac{25}{32}m_s v_0^2 = 37.5\ 10^8 J$ while $U = -\frac{Gm_s m_e}{r_0} = -48\ 10^8\ J$ adding these we get for the total energy $E = -\frac{7}{32}m_s v_0^2 = -10.5\ 10^8 J$

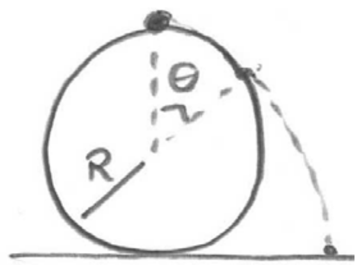
d) The total energy of the system after the rocket went off minus the total energy of the system before it went on is the total work done by the rocket that is:

$W = \frac{9}{32}m_s v_0^2 = 13.5\ 10^8 J$

e) For escaping to $\infty$the satellite must have total energy 0 that is $v = \sqrt{2}v_0 = 4.4\ 10^3 ms^{-1}$with this speed in fact $K = m_s v_0^2 = -U$

**Problem 2:** ***Conservation of Mechanical Energy and Newton's Second Law***

A small object of mass $m = 0.2\,kg$ is placed at the top of a large sphere of radius $R = 0.5\,m$ resting on the ground. The object is given a negligibly small velocity so that it starts to slide down the sphere. Assume the surface of the sphere is frictionless and the sphere is fixed to the surface of a table. In this problem, you will try to find where the object hits the ground.




a) Briefly describe how you intend to model the motion. Here are some questions which should help guide your thinking. Is there any special condition(s) that describe when the object leaves the sphere? Does the normal force do any work on the object? Does the principle of Conservation of Energy replace Newton's Second Law in the radial direction or are they independent?

b) At what angle will the mass leave the sphere?

**c)** What is the velocity of the mass when it just leaves the sphere?

**d)** How far from the bottom of the sphere will the mass hit the ground?

a) The object leaves the sphere when the contact normal force between it and the sphere goes to 0.

After that the object is in free fall and we can apply the projectile motion rules. Before that we will use conservation of energy to find the speed of the object while sliding on the sphere. We will have to use Newton's second law to determine the normal force acting on the object from its acceleration.

b) First we have to compute the speed of the mass when it has slided for an angle $\theta$on the sphere. Using conservation of energy we find:

$v(\theta) = \sqrt{2gR(1-\cos(\theta)} = 2\sqrt{gR}\sin(\theta/2)$

Then the radial component of the acceleration is: $\vec{a} \cdot \hat{r} = -\frac{v^2}{R} = -4g\sin(\theta/2)$

From Newton's second law the normal force must be: $|\vec{N}| = mg\cos(\theta) - 2gm(1-\cos(\theta))$

notice that this equation is only valid before the mass leaves the sphere.

Imposing $N = 0$ we finf the angle at which the mass leaves the sphere:$\theta = \arccos(2/3) = 0.27\pi$. Notice that this point is at an height $h = \frac{5}{3}R = 0.83\ m$ above the ground.

c) Using the previous formula for the speed we get $|v| = \sqrt{\frac{2}{3}gR} = 1.8\ ms^{-1}$this speed will have an horizontal component equal to $|v_h| = 1.2\ ms^{-1}$ and a vertical component $|v_v| = 1.34\ ms^{-1}$

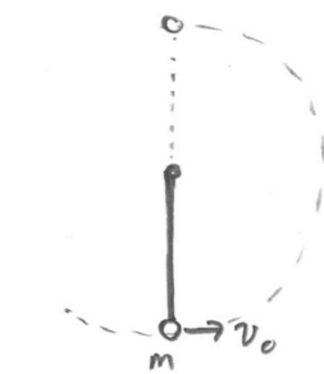
d) After leaving the sphere the mass will fly for a time $t$ such that:

$\frac{1}{2}gt^2 + |v_v|t - h = 0$ that is $t = g^{-1}(-|v_v| + \sqrt{v_v^2 + 2gh}) = 0.30\ s$

In this time the mass covers the horizontal distance: $d = |v_h|\ t = .36\ m$ and so touches the ground at a distance $\frac{\sqrt{5}}{3}R + d = 0.73\ m$ from the bottom of the sphere.

**Problem 3:** ***Conservation of Energy and Newton's Second Law: Tetherball***

A ball of negligible radius and mass $m = 0.1 kg$ hangs from a string of length $l = 0.5 m$. It is hit in such a way that it then travels in a vertical circle (with negligible loss of energy). The initial speed of the ball after being struck is $v_0 = 7.0 m/s$.

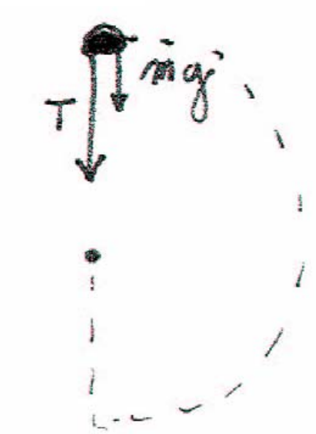



a) Find the speed of the ball at the top of the circle.

b) Find the tension in the string when the ball is at the top of the circle.

a)Using conservation of energy we have $\frac{1}{2}mv_{top}^2 = \frac{1}{2}mv_0^2 - 2lgm$ so that $v_{top} = \sqrt{v_0^2 - 4lg} = 5.4\ ms^{-1}$

b) The tension of the string is found by applying Newton's second law: $mg + T = m\frac{v_{top}^2}{l}$ that is $T = m\frac{v_0^2}{l} - 5mg = 4.9\ N$



**Problem 4:** ***Spring-mass harmonic oscillator***

Consider an ideal spring that has an unstretched length $l_0$. Assume the spring has a constant $k$. Suppose the spring is attached to a mass $m$ that lies on a horizontal frictionless surface. The spring-mass system is compressed a distance of $x_0$ from equilibrium and then released with an initial speed $v_0$ toward the equilibrium position.

a) What is the period of oscillation for this system?

b) How long will it take for the mass to first return to the equilibrium position?

c) How long will it take for the mass to first become completely extended?

d) Draw a graph of the position and velocity of the mass as a function of time. Carefully label your axes and clearly specify any special values.

## Problem 4

a) By solving the equation of motion $m\ \ddot{x} = -kx$ with the initial conditions $x(0) = -x_0$and $\dot{x}\ (0) = v_0$ we get $x(t) = -A\cos(\sqrt{\frac{k}{m}}\ t+\phi)$ with the parameters $A$and $\phi$such that:

$$A\cos(\phi) \quad = \quad x_0$$

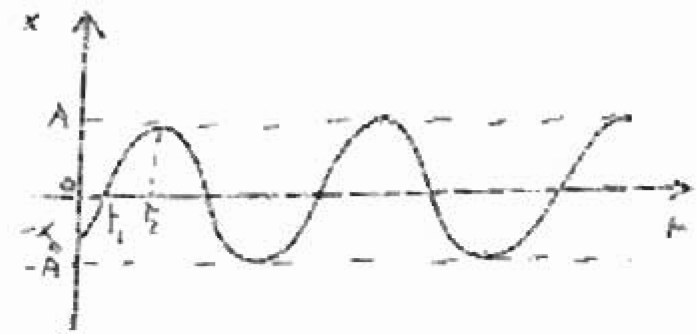
$$A\sqrt{\frac{k}{m}} \quad \sin(\phi) = \quad v_0$$

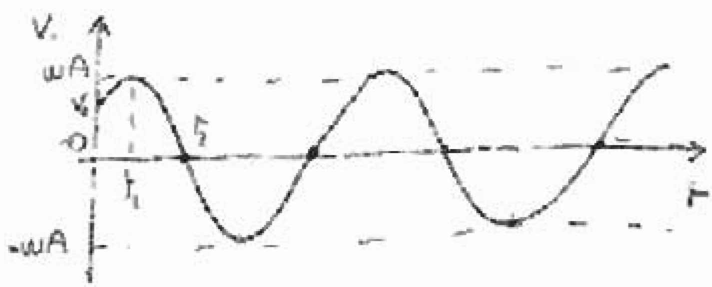
from which $\tan(\phi) = \frac{v_0}{x_0}\sqrt{\frac{m}{k}}$ and $A = \sqrt{x_0^2 + \frac{m}{k}v_0^2}$

The period is given by: $T = 2\pi\sqrt{\frac{m}{k}}$

b)the mass will return to the equilibrium position after a time $t_1$ such that $t_1\sqrt{\frac{k}{m}} = \frac{\pi}{2} - \phi$ that is $t_1 = \sqrt{\frac{m}{k}}(\frac{\pi}{2} - \phi) = T(\frac{1}{4} - \frac{\phi}{2\pi})$

c) The mass will be completely extended after a time $t_2$ such that $t_2\sqrt{\frac{k}{m}} = \pi - \phi$ that is $t_2 = \sqrt{\frac{m}{k}}(\pi - \phi) = T(\frac{1}{2} - \frac{\phi}{2\pi})$



**Problem 5: Analysis Experiment 06 Post**

a) Enter the values from your experiment into the table below.

| $a_{up}$ (m/s$^2$) | $a_{down}$ (m/s$^2$) | $U(h_1)$ (mJ) | $U(h_2)$ (mJ) |
|---|---|---|---|
| | | | |

Use these numbers, along with $M$ = 0.75 kg, to calculate the friction force on the cart. Compare it with the value you obtained from the potential energy difference between the turning points $h_1$ and $h_2$.

b) Explain briefly in words why it makes sense to fit the function $A \sin[2\pi(x\text{-}C)/T]$ to the force peaks when the cart bounces off the spring on the force sensor.

c) Enter the results of your fit to the second bounce into this table:

| $A$ (N) | $k$ (N/m) | $A^2$/2k (mJ) |
|---|---|---|
| | | |

and use them to find the force constant of the spring. Compute the potential energy stored in the compressed spring, $A^2/2k$, and compare it with the average of $U(h_1)$ and $U(h_2)$ from the table above.

a) Enter the values from your experiment into the table below.

| Quantity: | $a_{up}$ (m/s$^2$) | $a_{down}$ (m/s$^2$) | $U(x_1)$ (mJ) | $U(x_2)$ (mJ) |
|---|---|---|---|---|
| My value: | $0.390 \pm 0.008\,\mathrm{m/s^2}$ | $0.330 \pm 0.005\,\mathrm{m/s^2}$ | 61.9 mJ | 51.8 mJ |

**Answer:** any sensible numbers are fine.

Use these numbers, along with $M = 0.75\,\mathrm{kg}$, to calculate the friction force on the cart. Compare it with the value you obtained from the potential energy difference between the turning points $x_1$ and $x_2$.
**Answer:** from the free body diagrams, it is clear that $F_{\mathrm{friction}} = M(a_{\mathrm{up}} - a_{\mathrm{down}})/2 = 22.5\,\mathrm{mN}$. Between $x_1$ and $x_2$ cart travelled 0.45 m, and $U(x_1) - U(x_2) = 10.1\,\mathrm{mJ}$. Thus $F_{\mathrm{friction}} = 10.1/0.45 = 22.4\,\mathrm{mN}$.

b) Explain briefly in words why it makes sense to fit the function $A\sin[2\pi(x - C)/T]$ to the force peaks when the cart bounces off the spring on the force sensor.
**Answer:** anything that captures the idea that it is $\frac{1}{2}$ of a harmonic oscillator is OK.

c) Enter the results of your fit to the second bounce into this table

| Quantity: | $A$ (N) | $T$ (s) | $k$ (N/m) | $A^2/2k$ (mJ) |
|---|---|---|---|---|
| My values: | $15.3 \pm 0.12\,\mathrm{N}$ | $0.130 \pm 0.001\,\mathrm{s}$ | | |

and use them to find the force constant of the spring.
**Answer:** $\omega = 2\pi/T = \sqrt{k/m}$, thus $k = 4\pi^2 m/T^2 = 1750\,\mathrm{N}$.

Compute the potential energy stored in the compressed spring, $A^2/2k$, and compare it with the average of $U(x_1)$ and $U(x_2)$ from the table above.
**Answer:** $U = \frac{1}{2}kx^2_{\mathrm{max}} = A^2/2k = 66.8\,\mathrm{mJ}$, while $[U(x_1)$ and $U(x_2)]/2 = 59.8\,\mathrm{mJ}$. Should be closer, but that's what I got.