Problem 1. Rotational Kinematics.

a) In 2004, the period of rotation is $T_E = 86164\,s$. Thus the angular velocity is

$$\omega_E = \frac{2\pi}{T_E} \approx 7\,10^{-5}\,s^{-1}$$

The radius of the circle of rotation of the person at MIT is

$$r = R_E \cos\theta \approx 4.71\,10^6\,m.$$

Hence the linear velocity is

$$v = \omega_E r = \frac{2\pi}{T_E} R_E \cos\theta \approx 343.7\,m/s$$

and the centripetal acceleration is

$$a = \omega_E^2 r \approx 0.025 m/s^2.$$

b) The average angular deceleration is

$$\left\langle \frac{d\omega_E}{dt} \right\rangle = \frac{\Delta\omega_E}{\Delta t} = \frac{2\pi}{T_{2004} T_{1924} \Delta t} \left(T_{2004} - T_{1924}\right) \approx \frac{2\pi}{T_{2004}^2 \Delta t} \left(T_{2004} - T_{1924}\right)$$

where T_{2004} and T_{1924} refer to the period of the Earth's rotation in 2004 in 1924 and $\Delta t = 80$ years. Substituting numbers, we obtain

$$\left\langle \frac{d\omega_E}{dt} \right\rangle \approx 9\,10^{-22}\,s^{-2}$$

Problem 2. Second Law Applications.

a) The force diagram is shown on Fig. 1.

b) Using the coordinate system shown on Fig. 1, we get

$$\begin{aligned} r: &\quad T_1 \sin\theta + T_2 \sin\theta = m a_r = m\omega^2 l \sin\theta \\ y: &\quad T_1 \cos\theta - T_2 \cos\theta - mg = 0, \end{aligned}$$

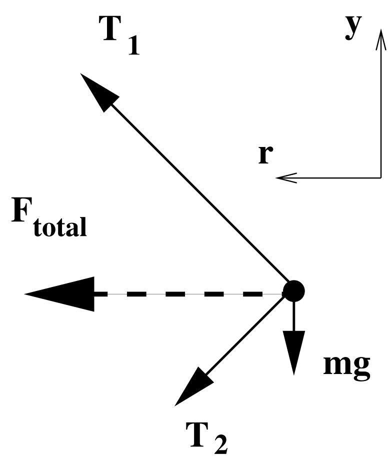

Figure 1: Force diagram for Problem 2

where $\theta = 45°$ and $\sin\theta = \cos\theta = 1/\sqrt{2}$. Thus

$$T_1 = \frac{1}{2}m\left(\omega^2 l + \sqrt{2}g\right)$$
$$T_2 = \frac{1}{2}m\left(\omega^2 l - \sqrt{2}g\right)$$

Problem 3. Static Equilibrium.

When the beam is in equilibrium, the sum of all forces and the sum of all torques should be zero. Considering the torque about the pivot (see Fig. 2) and introducing the coordinate system with y pointing up and x pointing to the right, we have:

$$x: \; F_x - T\cos\phi = 0$$
$$y: \; F_y + T\sin\phi - mg = 0$$

and

$$\text{torque:} \quad mg\frac{l}{2}\cos\theta - Th\cos\phi = 0$$

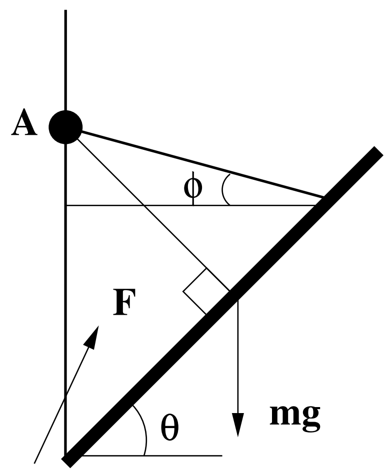

Figure 2: Beam supported by a cable

where T is the tension in rope, F_x and F_y denote x and y component of $\vec{F}$ and we introduced two additional parameters — the height h of the point where the rope is connected to the pole and angle ϕ (see Fig. 2).

To solve the above equations, we note that the torque equation gives

$$T = mg\frac{l}{2h}\frac{\cos\theta}{\cos\phi}$$

thus

$$F_x = mg\frac{l}{2h}\cos\theta$$
$$F_y = mg - mg\frac{l}{2h}\cos\theta\tan\phi.$$

From the geometry of the problem

$$\tan\phi = \frac{h - 2/3\,l\sin\theta}{2/3\,l\cos\theta}$$

Thus

$$F_y = mg\left(\frac{1}{4} + \frac{l}{2h}\sin\theta\right)$$

Thus the magnitude of the force is

$$F = \sqrt{F_x^2 + F_y^2} = \frac{mg}{4}\sqrt{1 + \frac{4l^2}{h^2} + \frac{4l}{h}\sin\theta}$$

and is increased when θ is increased.

Problem 4. Experiment 4 Data Analysis.

We use the following data:

T	r_m	ω	ΔX	$m_m r_m \omega^2$
∞	$4.8\,cm$	$0\,s^{-1}$	$0\,m$	$O\,N$
$0.095\,s$	$6.0\,cm$	$66.1\,s^{-1}$	$0.012\,m$	$2.44\,N$
$0.074\,s$	$7.2\,cm$	$84.9\,s^{-1}$	$0.024\,m$	$4.83\,N$
$0.060\,s$	$10.3\,cm$	$105\,s^{-1}$	$0.055\,m$	$10.5\,N$

a) Our fit gave $A = 4.7\,cm$, which is close (within the measurement error) to $r_0 = 4.8\,cm$.

b) We get a better fit when A is an adjustable parameter (root mean square error is $\sigma = 0.07\,cm$ vs $\sigma = 0.14\,cm$ when it is not adjustable — see Fig. 3).

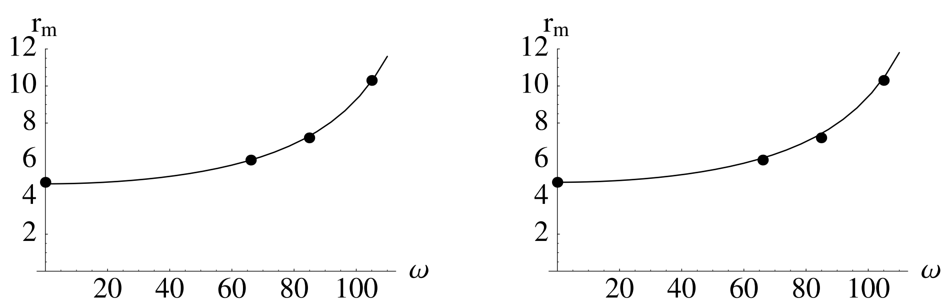

Figure 3: The one and two-parameter fits. Left: two-parameter fit. $A = 4.7\,cm$, $\omega_c = 143\,s^{-1}$, RMSE is $0.07\,cm$. Right: one-parameter fit, $\omega_c = 144\,s^{-1}$, RMSE is $0.14\,cm$.

c) The one-parameter fit gives $\omega_c = 144\,s^{-1}$. On the other hand, $\sqrt{k/m} = 142\,s^{-1}$.

d) The root mean square error of our fit, $\sigma = 0.07\,cm$, is of the order of the measurement error, which is about $1\,mm$.