Problem 1. Measurement of g.

a) The ball moves in the vertical direction under the influence of the constant force of gravity[1]. Hence in our approximation the ball undergoes one-dimensional motion with constant acceleration g. Let us introduce vertical axis x and choose the positive direction to point up. We then place the origin of the coordinate system at the bottom of the lower pane of glass and choose to measure the time from the moment when the ball rises past the origin, i.e. we choose $x_0 = 0$ and $t_0 = 0$ (See Fig. 1). With our choice of initial position

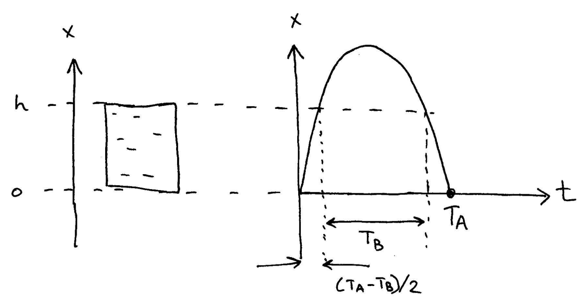

Figure 1:

and time, the equation for $x(t)$ is

$$x(t) = v_0 t - \frac{gt^2}{2} \tag{1}$$

and the graph of $x(t)$ should look like the one on Fig. 1 The velocity of the ball is given by

$$v(t) = v_0 - gt \tag{2}$$

b) Using Fig. 1, it is easy to understand that the time the ball travels across the window, i.e. the interval of time between the moment it rises past the bottom of the lower plane of glass and disappears past the top of the upper pane, is $\tau = (T_A - T_B)/2$. Since the height of the glass is h, we obtain, from Eq. (1)

$$h = v_0 \tau - \frac{g\tau^2}{2} \tag{3}$$

[1] We neglect the air resistance and the variation of the gravitational force over the height of the ball's trajectory

To find the second equation, we notice that the highest point of the trajectory corresponds to $v = 0$ and $t = T_A/2$. Thus we can find v_0 as

$$v_0 = gT_A/2 \tag{4}$$

We now substitute this result back into Eq. (3) and obtain

$$g = \frac{2h}{\tau(T_A - \tau)} = \frac{8h}{T_A^2 - T_B^2} \tag{5}$$

Problem 2. Track event.

a) Let us place the origin of the coordinate system at 48 m before the finish line, which is the position of Jim when he starts accelerating, and measure the time from the moment Jim passes this point. Then Bob's initial position is $x_0 = 2$ m and the diagram of $x(t)$ should look like the one shown on Fig.2.

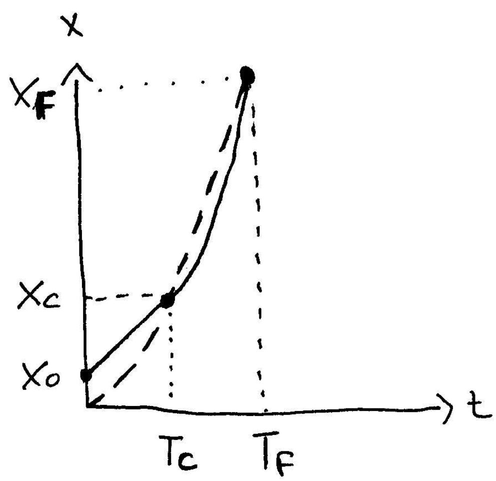

Figure 2: Position of the runners vs time. Solid line: position of Bob; dashed line: position of Jim

We can identify two distinct phases of motion:
Phase I: for $0 < t \leq t_C$ Jim accelerates with constant acceleration $a_J =$

1 m/s while Bob moves with constant velocity v_0=8 m/s.
Phase II: for $t_C < t \leq t_F$ Jim runs with constant velocity v_{J2} while Bob accelerates with constant acceleration a_B.

The position of Jim during Phase I is described by

$$x_J(t) = v_0 t + \frac{a_J t^2}{2} \tag{6}$$

and his velocity is

$$v_J(t) = v_0 + a_J t \tag{7}$$

The position and velocity of Bob during Phase I are given by

$$x_B(t) = x_0 + v_0 t \tag{8}$$

$$v_B(t) = v_0 \tag{9}$$

b) When Jim catches up to Bob, $x_B = x_J$. Hence we find that this happened at

$$t = t_C = \sqrt{2x_0/a_J} = 2s. \tag{10}$$

c) We can use either (8) or (6) together with (10) to find the position of both runners when Jim catches up to Bob. It is

$$x_C = x_0 + v_0 t_C = 18\,m \tag{11}$$

Hence both runners are $x_F - x_C = 48\,m - 18\,m = 30$ m away from the finish line.

d) After Jim caught up to Bob, he moves with constant velocity

$$v_{J2} = v_0 + a_J t_C = 10\,m/s \tag{12}$$

Therefore, it will take him $(x_F - x_C)/v_{J2} = 3$ s to reach the finish.

e) During Phase II, Bob's position is given by

$$x_{B2}(t) = x_C + v_0(t - t_C) + \frac{a_B(t - t_C)^2}{2} \tag{13}$$

At the finish line $t = t_F$ and $x_B = x_F$. Using the above equation, we obtain Bob's acceleration during Phase II:

$$a_B = 2\frac{(x_F - x_C) - v_0(t_F - t_C)}{(t_F - t_C)^2} \approx 1.3 \quad m/s^2 \tag{14}$$

f) Bob's velocity at the finish line is

$$v_B(t_F) = v_0 + a_{B2}(t_F - t_C) = 12 \quad m/s. \tag{15}$$

Bob is running faster at the finish line. This can also be seen easily from the diagram on Fig 2.

Problem 3. Projectile motion. Softball.

a) We use a Cartesian coordinate system with the origin located above the home plate at the point where the ball leaves the bat. Let axis x point towards the third base and the axis y point upwards. In this coordinate system the motion along x is decried by a pair of equations

$$v_x(t) = v_{x0} \qquad\qquad x(t) = v_{x0}t \tag{16}$$

and the motion along y is described by

$$v_y(t) = v_{y0} - gt \qquad\qquad y(t) = v_{y0}t - \frac{gt^2}{2} \tag{17}$$

where g is the gravitational acceleration.

b) The ball reaches the highest point on its trajectory at $t = \Delta t/2$. Using the equation for $v_y(t)$, we find

$$v_{y0} = g\Delta t/2. \tag{18}$$

The distance that the ball travels along x before the third baseman catches it, is $d_1 + v_1\Delta t$. Thus we find

$$v_{x0} = \frac{d_1 + v_1\Delta t}{\Delta t}.$$

Hence the initial angle was

$$\tan\theta_0 = \frac{v_{y0}}{v_{x0}} = \frac{g\Delta t^2}{2(d_1 + v_1\Delta t)} \approx 0.55 \approx 31.5^\circ \tag{19}$$

and the initial velocity was

$$v_0 = \sqrt{v_{x0}^2 + v_{y0}^2} = 18.8\,m/s \tag{20}$$

c) The moment of time corresponding to 0.1 s before the ball was caught is $t = \Delta t - 0.1$ s. Using (16) and (17), we obtain

$$x = 30.4\,m \qquad v_x = v_{x0} = 16 m/s \tag{21}$$

$$y = 0.9\,m \qquad v_y = -8.8\,m/s, \tag{22}$$

which can be written in vector form as

$$\vec{r} = (30.4\vec{\imath} + 0.9\vec{\jmath})\ m \tag{23}$$

and

$$\vec{v} = (16\vec{\imath} - 8.8\vec{\jmath})\ m/s. \tag{24}$$

Here $\vec{\imath}$ and $\vec{\jmath}$ are vectors of unit length in the direction of x and y correspondingly.

Problem 4. Pre-Lab question.

a) Since $\ln|\vec{F}| = \ln a - bx$, the tangent of slope of the graph gives constant b: $\tan\theta = -b$.

b) This time $\ln|\vec{F}| = \ln c + d\ln x$. Hence d can be obtained from the slope of the graph of $\ln|\vec{F}|$ vs $\ln x$.

Problem 5. Experiment 2 Data analysis.

Part A We performed the projected ball experiment. The tube was set at the angle $\theta_0 = 30^\circ$ and the exit point was $1.21\,m$ above the ground. We estimate the uncertainty in the measurement of hight to be $2\,mm = 2\,10^{-3}\,m$

and in the measurement of the horizontal distance to be $5\,10^{-3}m$. Although is is somewhat difficult to estimate the uncertainty in the measurement of angle, it is likely to be a few degrees. We take $\delta\theta_0 = 1°$. The error in the measurement of the pulse width ΔT is about 10% of the measured value. The results of the experiment are summarized in the table below. Here d is the horizontal distance, v_{0T} is the calculated exit velocity based on theory, ΔT is the measured pulse width and v_{0C} is the calculated exit velocity based on the pulse width.

Trial	d,m	$v_{0T}, m/s$	ΔT, s	$v_{0C}, m/s$
1	0.777	1.4	$8.3\,10^{-3}$	1.5
2	0.812	1.5	$8.0\,10^{-3}$	1.6
3	0.772	1.5	$8.3\,10^{-3}$	1.5
Mean	0.787	1.5	$8.1\,10^{-3}$	1.5

We see that the exit velocity calculated using the theory agrees fairly well with the value measured in the experiment.

Part B We use the same measurements as in Part A, but this time calculate the gravitational constant g using the value of the initial velocity calculated from our measurements of the pulse width, v_{0C}

Trial	d,m	ΔT, s	$v_{0C}, m/s$	$g\,m/s^2$
1	0.777	$8.3\,10^{-3}$	1.5	8.7
2	0.812	$8.0\,10^{-3}$	1.6	8
3	0.772	$8.3\,10^{-3}$	1.5	8.8
Mean	0.787	$8.1\,10^{-3}$	1.5	8.5

We note that an error of about 10% in the measurement of ΔT introduces the same fractional error in the calculation of v_0. Indeed,

$$\left|\frac{\Delta v_{0C}}{v_{0C}}\right| = \frac{D}{\Delta T^2}\delta\Delta T = 0.1\frac{D}{\Delta T} = 0.1 \tag{25}$$

Since g depends quadratically on v_{0C}, the fractional error in g introduced by the error in v_{0C} is

$$\frac{\Delta g}{\bar{g}} \approx 2\frac{\Delta v_{0C}}{v_{0C}} \approx 0.2 \tag{26}$$