

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Physics

Physics 8.01 TEAL

Fall Term 2004

**In-Class Problems 23-24: Harmonic Oscillation and Mechanical Energy**

Section \_\_\_\_\_ Table and Group Number \_\_\_\_\_

Names \_\_\_\_\_

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\_\_\_\_\_

**Hand in one solution per group.**

**Both these problems today are hard. We would like you to concentrate your discussions on the physics ideas that enable you to set-up and plan a strategy for solving these problems. We may interrupt the discussions to discuss the problems.**

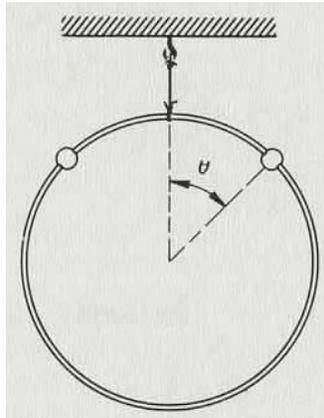
## Problem 22: Journey to the Center of the Earth

Imagine that one drilled a hole with smooth sides straight through the center of the earth. If the air is removed from this tube (and it doesn't fill up with water, liquid rock, or iron from the core) an object dropped into one end will have enough energy to just exit the other end after an interval of time. Your goal is to find that interval of time.

- a) The gravitational force on an object of mass  $m$ , located inside the earth a distance  $r < R_e$ , from the center ( $R_e$  is the radius of the earth), is due only to the mass of the earth that lies within a solid sphere of radius  $r$ . What is the gravitational force as a function of the distance  $r$  from the center? Express your answer in terms of  $g$  and  $R_e$ . Note: you do not need the mass of the earth to answer this question. You only need to assume that the earth is of uniform density.
- b) Using the concept of force explain how the object-earth system for objects inside the earth is analogous to an object-spring system.
- c) What is the potential energy inside the earth as a function of  $r$  for the object-earth system? Can you think of a natural point to choose a zero point for the potential energy? Find out how long this journey will take.
- d) How does this time compare to a satellite in low earth orbit (i.e. orbits just above the surface of the earth)?
- e) In order to shorten the travel time, the sender travels down the hole a distance until she is only a fraction of the radius of the earth,  $\beta R_e$ , ( $R_e$  is the radius of the earth) from the center. The recipient travels down a similar distance from the far end in order to catch the object. By how much if any is the travel time thereby reduced (as a function of  $\beta$ )?

### Problem 23: Beads Sliding along a Ring

A ring of mass  $m_1$  hangs from a thread, and two identical beads of mass  $m_2$  slide on it without friction. The beads are released simultaneously from the top of the ring from rest (actually they need a very small initial speed but this can be ignored) and slide down opposite sides. Assume  $m_2 > (3/2) m_1$ . Then ring will start to rise when the beads reach a critical angle  $\theta_c$  with respect to the vertical.



- Draw free body force diagrams for the ring and the beads. What direction is the force of the bead on the ring pointing? Does it change as the bead moves? Can you still proceed with an analysis using Newton's Second Law if you are not sure which way this force points? Try to find a physical explanation for the direction of this force. What is the condition that the ring just starts to rise?
- Does the mechanical energy change between when the beads are released and the ring just starts to rise? Write down an equation that describes the changes in the mechanical energy.
- This is hard!** Can you show that the ring will start to rise if  $m_2 > (3/2) m_1$ ? Find the angle  $\theta_c$  with respect to the vertical direction that this occurs.