**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**
**Department of Physics**

**Physics 8.01 TEAL** **Fall Term 2004**

# In-Class Problems 27-29: Momentum and Collisions: Solutions

## Problem 27: Elastic One Dimensional Collision

Consider the elastic collision of two carts along a track; the incident cart 1 has mass $m_1$ and moves with initial velocity $v_{1,0}$. The target cart has mass $m_2 = 2m_1$ and is initially at rest $v_{2,0} = 0$. Immediately after the collision, the target cart has final speed $v_{1,f}$ and the target cart has final speed $v_{2,f}$. Calculate the final velocities of the carts as a function of the initial velocity $v_{1,0}$.

**Solution**
a) Draw a momentum diagram.

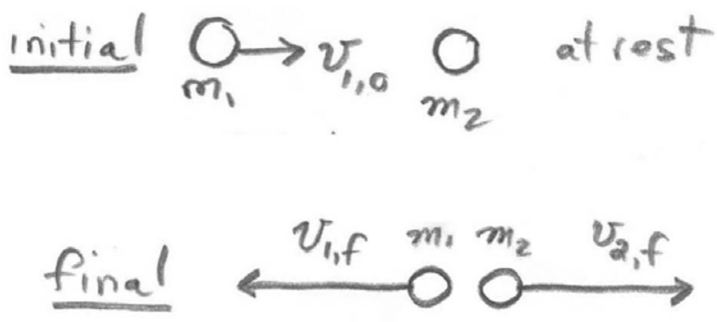



**Figure 10.11** Momentum flow for a one-dimensional collision.

b) Find the equation for conservation of momentum in the lab reference frame.

$$m_1 v_{1,0} = -m_1 v_{1,f} + 2m_1 v_{2,f}\,.$$

Thus

$$v_{1,0} = -v_{1,f} + 2v_{2,f}\,.$$

c) Find the equation for conservation of energy in the lab reference frame.

$$\frac{1}{2} m_1 {v_{1,0}}^2 = \frac{1}{2} m_1 {v_{1,f}}^2 + \frac{1}{2} 2m_1 {v_{2,f}}^2\,.$$

Thus

$$v_{1,0}{}^{2} = v_{1,f}{}^{2} + 2v_{2,f}{}^{2}.$$

d) Using these two equations, calculate the final velocities of the target cart and the incident cart.

We shall solve for $v_{2,f}$. From the momentum equation,

$$v_{1,f} = 2v_{2,f} - v_{1,0}.$$

The energy equation becomes

$$v_{1,0}{}^{2} = \left(2v_{2,f} - v_{1,0}\right)^{2} + 2v_{2,f}{}^{2} = 4v_{2,f}{}^{2} + v_{1,0}{}^{2} - 4v_{2,f}v_{1,0} + 2v_{2,f}{}^{2}.$$

Thus

$$0 = 6v_{2,f}{}^{2} - 4v_{2,f}v_{1,0}.$$

We can now solve for $v_{2,f}$. There are two solutions. The first is the trivial $v_{2,f} = 0$, which is just the initial state when the target is at rest. The reason we get this solution is that the energy and momentum equations are the same if we reverse the direction of time (change the direction of all velocities). The second solution is the one of interest,

$$v_{2,f} = \frac{2}{3}v_{1,0}.$$

Then the final velocity of the incident cart is

$$v_{1,f} = 2v_{2,f} - v_{1,0} = \frac{4}{3}v_{1,0} - v_{1,0} = \frac{1}{3}v_{1,0}.$$

**Problem 28:** ***(Conservation of Momentum)***

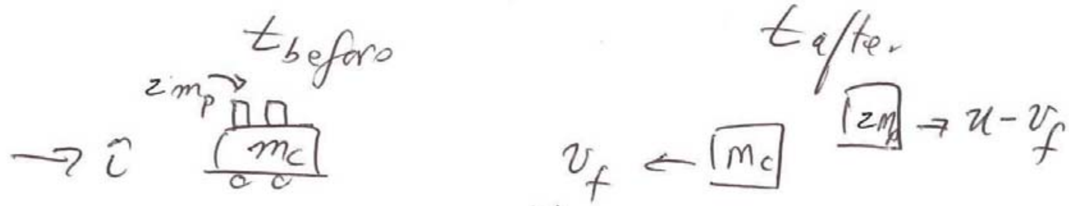
Two people, each with mass $m_p$, stand on a railway flatcar of mass $m_c$. They jump off one end of the flatcar with velocity of magnitude $u$ relative to the car. The car rolls in the opposite direction without friction.

a) What is the final velocity of the car if the two people jump at the same time?

b) What is the final velocity of the car if the two people jump off one at a time?

c) In which of the two above cases, does the railcar have the greatest final velocity?

a) choose reference frame at rest with respect to ground




$v_b = 0$

$\vec{P}_{before} = 0\ \hat{i}$ $\qquad$ $\vec{P}_{after} = (2m_p(u - v_f) - m_c v_f)\ \hat{i}$

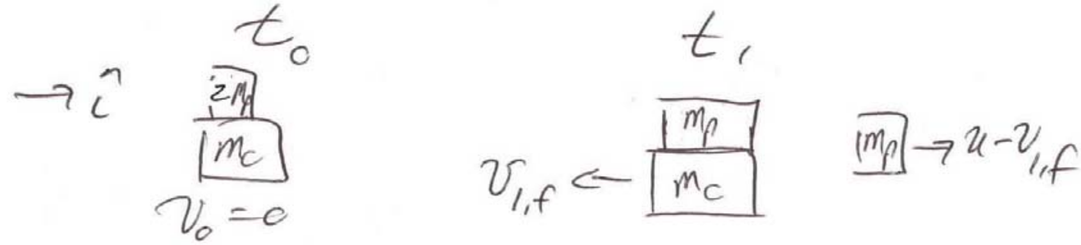
| | $\Delta t \vec{F}_{ext}$ | $= \Delta \vec{P}$ |
|---|---|---|
| $\hat{i}:$ | $0$ | $= P_{after} - P_{before}$ |

$$0 = 2m_p(u - v_f) - m_c v_f$$

solve for $v_f$: $\quad v_f = \dfrac{2m_p u}{2m_p + m_c}$

note: $\quad v_f = \dfrac{m_{people}^{total}\, u}{m_{people}^{total} + m_{cart}^{total}} = \dfrac{m_{people}^{total}\, u}{m^{total}}$

b) Suppose the people jump off one at a time: in ground frame:



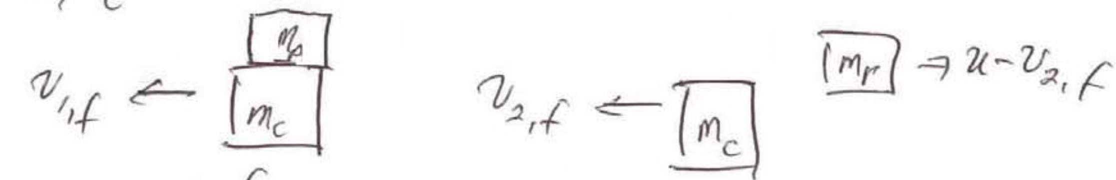
$\hat{i}: \quad 0 = P_1 - P_0 = m_p(u - v_{1,f}) - (m_c + m_p)v_{1,f}$

solve for $v_{1,f}$:

$$v_{1,f} = \frac{m_p u}{m_c + 2m_p} = \frac{m_{person}\, u}{m_{total}}$$

Second person jumps off: ground reference frame

$\rightarrow \hat{i}$




befo

$$\hat{i}: \quad 0 = P_2 - P_1 = (m_p(u - v_{2,f}) - m_c v_{2,f}) - (-(m_c + m_p)v_{1,f})$$

solve for $v_{2,f}$:

$$m_p u + (m_c + m_p)v_{1,f} = (m_c + m_p)v_{2,f}$$

$$v_{2,f} = \frac{m_p u}{m_c + m_p} + v_{1,f}$$

$$v_{2,f} = \frac{m_p u}{m_c + m_p} + \frac{m_p u}{m_c + 2m_p}$$

c) Compare the result from part b) to the result from part a)

if they all jump off at once.

$$v_f = \frac{2m_p u}{2m_p + m_c} = \frac{m_p u}{2m_p + m_c} + \frac{m_p u}{2m_p + m_c}$$

When they jump off one at a time,

$$v_{2,f} = \frac{m_p u}{m_c + m_p} + \frac{m_p u}{m_c + 2m_p}$$

comparing these expressions, note

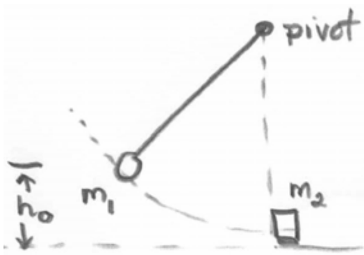
$$\frac{m_p u}{2m_p + m_c} < \frac{m_p u}{m_c + m_p}$$

Therefore $v_f < v_{2,f}$. The velocity of the cart is slower if they all jump off at once.

The explanation is that when they jump off at once, they are pushing the entire mass of cart and all the people. When they jump off one at a time, each successive person has to push a slightly lighter cart (less people) so the cart recoils faster.

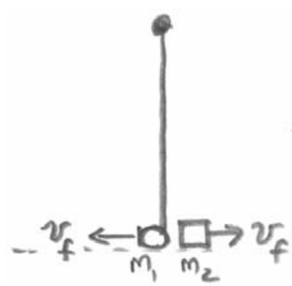
**Problem 29:** ***Pendulums and Collisions***

A simple pendulum consists of a bob of mass $m_1$ that is suspended by a massless string. The bob is pulled out and released from a height $h_0$ as measured from the bottom of the swing and swings downward in a circular orbit. At the bottom of the swing, the bob collides with a block of mass $m_2$ that is initially at rest on a frictionless table. Assume the pivot point is frictionless.



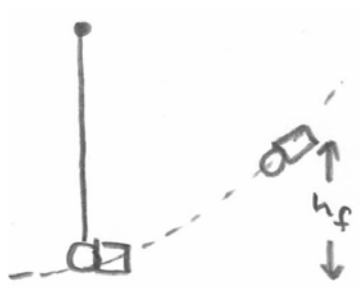

a) What is the velocity of the bob immediately before the collision at the bottom of the swing?

b) Assume the collision is perfectly elastic. The block moves along the table and the bob moves in the opposite direction but with the same speed as the block. What is the mass, $m_2$, of the block?




c) Suppose the collision is completely inelastic due to some putty that is placed on the block. What is the velocity of the combined system immediately after the collision? (Assume that the putty is massless.)

d) After the completely inelastic collision, the bob and block continue in circular motion. What is the maximum height, $h_f$, that the combined system rises after the collision?



The speed of the bob considered a point mass can be found applying conservation of energy:

$$v_0 = \sqrt{2 g h_0}$$

Applying conservation of momentum and conservation of energy we have

$$m_1 v_0 = (m_2 - m_1) v_f \qquad m_1 v_0^2 = (m_1 + m_2) v_f^2$$

$$\Rightarrow \quad m_1 v_0^2 = \frac{(m_1 + m_2)}{(m_1 - m_2)^2} m_1^2 v_0^2 \quad \Rightarrow \quad m_2 (m_2 - 3 m_1) = 0$$

The physically sensible solution is $m_2 = 3 m_1$ (The other one will have $v_f < 0$) In this case $v_f = v_0/2$

In the inelastic case $m_1 v_0 = (m_2 + m_1) v_f'$

$$v_f' = v_0 \frac{m_1}{m_2 + m_1} = \frac{v_0}{4}$$

Applying conservation of energy after the collision we find

$$(m_1 + m_2) g h_{max} = \frac{1}{2} (m_1 + m_2) v_f'^2 \Rightarrow \quad h_{max} = \frac{v_0^2}{32 g} = \frac{1}{16} h_0$$