

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 TEAL

Fall Term 2004

In-Class Problems 22-23: Mechanical Energy Solution

Section _____ Table and Group Number _____

Names _____

Hand in one solution per group.

We would like each group to apply the problem solving strategy with the four stages (see below) to answer the following two problems.

- I. Understand – get a conceptual grasp of the problem**
- II. Devise a Plan - set up a procedure to obtain the desired solution**
- III. Carry out your plan – solve the problem!**
- IV. Look Back – check your solution and method of solution**

Problem 22: *Escape Velocity and Mechanical Energy*

The asteroid Toro, discovered in 1964, has a radius of about $R = 5.0 \text{ km}$ and a mass of about $m_t = 2.0 \times 10^{15} \text{ kg}$. Let's assume that Toro is a perfectly uniform sphere. What is the escape velocity for an object of mass m on the surface of Toro? Could a person reach this speed (on Earth) by running?

Solution:

The only potential energy in this problem is the gravitational potential energy. We choose the zero point for the potential energy when the object and Toro are an infinite distance apart, $U_{gravity}(r_0 = \infty) \equiv 0$. Then the potential energy when the object and Toro are an infinite distance r apart is given by

$$U_{gravity}(r) = -\frac{Gm_t m}{r} \text{ with } U_{gravity}(r_0 = \infty) \equiv 0.$$

The expression *escape velocity* refers to the minimum velocity necessary for the object to escape the gravitational interaction of the asteroid and move off to an infinite distance away. If the object has a velocity less than the escape velocity, it will be unable to escape the gravitational force and must return to Toro. If it has a velocity greater than the escape velocity, it will have a non-zero kinetic energy at infinity. So the condition for the escape velocity is that the object will have exactly zero kinetic energy at infinity.

We choose our initial state, at time t_0 , when the object is at the surface of the planet with velocity equal to escape velocity. We choose our final state, at time t_f , to occur when the separation distance between the asteroid and the object is infinite.

Initial Energy: The initial kinetic energy is $K_0 = \frac{1}{2} m v_{esc}^2$. The initial potential energy is

$U_0 = -\frac{Gm_t m}{R}$. So the initial mechanical energy is

$$E_0 = K_0 + U_0 = \frac{1}{2} m v_{esc}^2 - \frac{Gm_t m}{R}.$$

Final Energy: The final kinetic energy is $K_f = 0$, since this is the condition that defines escape velocity. The final potential energy is zero, $U_f = 0$ since we chose the zero point for potential energy at infinity. So the final mechanical energy is

$$E_f = K_f + U_f = 0.$$

Non-conservative work: There is no non-conservative work.

Change in Mechanical Energy:

The change in mechanical energy

$$0 = W_{nc} = \Delta E_{mech},$$

is then

$$0 = \frac{1}{2}mv_{esc}^2 - \frac{Gm_i m}{R}$$

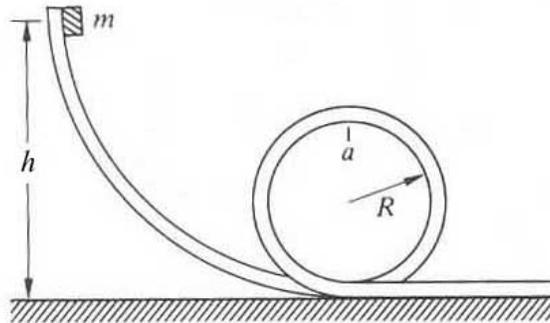
This equation can be solved for the escape velocity, v_{esc} ,

$$v_{esc} = \sqrt{\frac{2Gm_i}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 \cdot \text{kg}^{-2})(2.0 \times 10^{15} \text{ kg})}{(5.0 \times 10^3 \text{ m})}} = 7.3 \text{ m} \cdot \text{s}^{-1}$$

Considering that Olympic sprinters typically reach velocities of $12 \text{ m} \cdot \text{s}^{-1}$, this is an easy velocity to attain by running on Earth. It may be harder on Toro to generate the acceleration necessary to reach this speed by pushing off the ground, since any slight upward force will raise the center of mass and it will take substantially more time than on earth to come back down for another push off the ground.

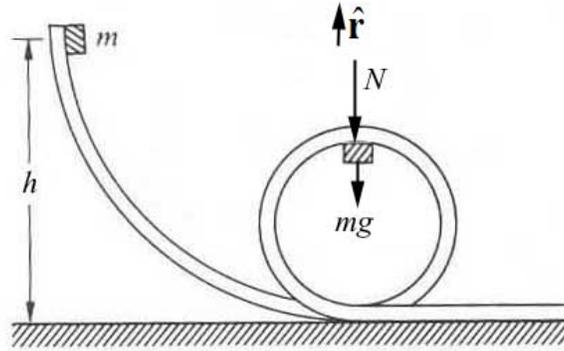
Problem 23: Circular Motion and Conservation of Mechanical Energy

An object of mass m is released from rest at a height h above the surface of a table. The object slides along the inside of the loop-the-loop track consisting of a ramp and a circular loop of radius R shown in the figure. Assume that the track is frictionless. When the object is at the top of the track it pushes against the track with a force equal to three times it's weight. What height was the object dropped from?



Solution:

We choose polar coordinates with origin at the center of the loop. We choose the zero point for the potential energy $U = 0$ at the bottom of the loop.



Initial Energy: We choose for our initial state, the instant the mass is released. The initial kinetic energy $K_0 = 0$. The initial potential energy is non-zero, $U_0 = mgh$. So the initial mechanical energy is

$$E_0 = K_0 + U_0 = mgh.$$

Final Energy: We choose for our final state, the instant the mass is at the top of the loop. The final kinetic energy $K_f = \frac{1}{2}mv_f^2$ since the mass is in motion. The final potential energy is non-zero, $U_f = mg2R$. So the final mechanical energy is

$$E_f = K_f + U_f = 2mgR + \frac{1}{2}mv_f^2.$$

Non-conservative Work: Since we are assuming the track is frictionless, there is no non-conservative work.

Change in Mechanical Energy: The change in mechanical energy is therefore zero,

$$0 = W_{nc} = \Delta E_{mechanical} = E_f - E_0.$$

Thus mechanical energy is conserved, $E_f = E_0$, or

$$2mgR + \frac{1}{2}mv_f^2 = mgh.$$

Missing Condition: The normal force of the track on the object is perpendicular to the direction of the motion of the object so this force does zero work,

$$\vec{N} \cdot d\vec{r} = 0.$$

Therefore the work-kinetic energy theorem does not account for the action of this force.

When there are forces that do no work in some direction, set up the Second Law in that direction,

$$\vec{F}_{\perp} = m\vec{a}_{\perp} .$$

We show the force diagram when the mass is at the top of the loop. Therefore Newton's Second Law in the radial direction, $\hat{\mathbf{r}}$, is

$$-mg - N = -mv_f^2 / R .$$

Notice from the information given in the example, the normal force of the loop on the object is $\vec{N} = -3mg\vec{r}$. (This is the action-reaction pair).

Therefore the Second Law becomes

$$4mg = mv_f^2 / R .$$

We can rewrite this condition in terms of the kinetic energy as

$$2mgR = \frac{1}{2}mv_f^2$$

Summary: Our two equations are therefore

$$2mgR + \frac{1}{2}mv_f^2 = mgh$$

$$2mgR = \frac{1}{2}mv_f^2 .$$

The second equation (from the Newton's Second Law) can be substituted into the conservation of mechanical energy equation to yield ,

$$4mgR = mgh .$$

So the initial height is

$$h = 4R .$$