MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01T **Fall Term 2004**

In-Class Problems 14-16: Uniform Circular Motion and Gravitation Solutions

Section_________Table and Group Number ______________________

Names ____________________________________

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Hand in one solution per group.

We would like each group to apply the problem solving strategy with the four stages (see below) to answer the following two problems.

I. Understand – get a conceptual grasp of the problem
II. Devise a Plan - set up a procedure to obtain the desired solution
III. Carry our your plan – solve the problem!
IV. Look Back – check your solution and method of solution

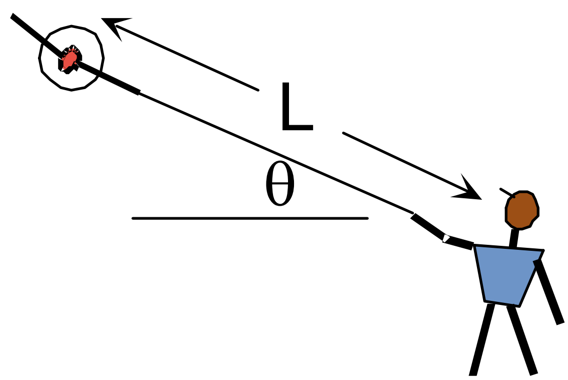
In-Class-Problem 14: Whirling Objects, U-Control Model Airplane

A U-control airplane of mass M is attached by wires of length L (and negligible mass) to the "pilot" who controls the lift provided by the wing. (The wires control the plane's elevator.) The plane's engine keeps it moving at constant speed v.

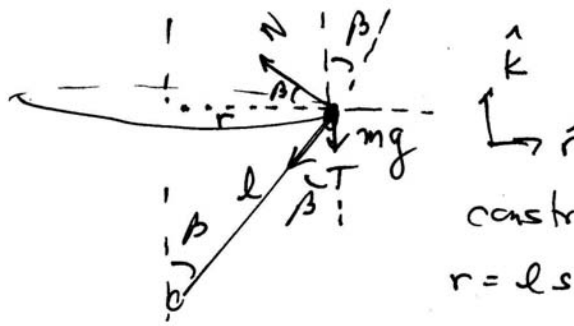
a) Briefly describe how you intend to model the motion of the object? What directions are you choosing for analyzing the components of your forces and state why?

b) Find the total tension T in the wires when the plane is flown overhead in a circle so that the wires make an angle θ with the ground. Remember that the wings can provide lift only in the direction perpendicular to their area, i.e. in a direction perpendicular to the wires. Think carefully before selecting the angle of your coordinate system.

c) The plane will go out of control and crash if the tension is not maintained. Given a particular speed of the plane, v, is there some angle θ_{crirt} which you would advise the pilot not to exceed? If possible, exhibit a speed vsafe, at which the plane would be safe at any angle?



Solution:



constraint

$r = l \sin\beta$

	$\vec{F}$	=	$m\vec{a}$
$\hat{r}:$	$-T\sin\beta - N\cos\beta$	=	$-\dfrac{mv^2}{r}$
$\hat{k}:$	$N\sin\beta - T\cos\beta - mg$	=	0

$$T\sin\beta + N\cos\beta = m r \omega^2 \quad (1)$$

$$N\sin\beta - T\cos\beta - mg = 0 \quad (2)$$

multiply eq (1) by $\sin\beta$, and eq (2) by $\cos\beta$

$$\sin\beta\,(T\sin\beta + N\cos\beta) = \frac{mv^2}{l\sin\beta}(\sin\beta)$$

$$-\cos\beta\,(N\sin\beta - T\cos\beta) = mg(-\cos\beta)$$

add $\quad T = \dfrac{mv^2}{l} - mg\cos\beta$

$T = 0$ when $\dfrac{mv^2}{l} = mg\cos$, string goes slack

so $v^2 > lg\cos\beta$

Notice that $\beta = \pi/2$ the rope lies in the plane of the airplane's circular obit. $\cos(\pi/2) = 0$, and the tension $T = mv^2/l > 0$ for all velocities. The other extreme value occurs when $\beta \to 0$. This correspond to the radius of the orbit $r \to 0$ Then $\cos(0) \to 1$ and the tension is $T \to (mv^2/l) - mg$. In order for the tension to stay positive $v > \sqrt{gl}$.

In-Class Problem 15: ***Uniform circular motion and the moon's period*)**

In this problem assume that the moon is only under the influence of the earth's gravitational force given by a magnitude $\vec{\mathbf{F}}_{e,m} = -G\dfrac{m_e m_m}{r_{e,m}^2}\hat{\mathbf{r}}_{e,m}$. Also assume that the moon is moving in a circular orbit around the earth and that the moon travels with a constant speed in that orbit. Let $G = 6.67\times10^{-11}\ N-m^2/kg^2$. The mass of the earth is $m_e = 5.98\times10^{24}\ kg$. The mass of the moon is $m_m = 7.36\times10^{22}\ kg$. The radius of the orbit is $r_{e,m} = 3.8\times10^8\ m$.

a) Briefly describe how you intend to model the motion of the object? What directions are you choosing for analyzing the components of your forces and state why?

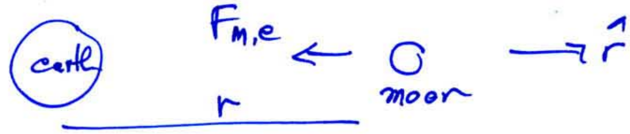
b) Calculate the period of the moon's orbit around the earth.

c) Is this the same period as the time between full moons as seen from the earth? Explain your reasoning.

Solution

Moon's orbit: circular motion, use radial unit vector, acceleration is inward $a_{rad} = -r\frac{4\pi^2}{T^2}$

Force diagram:



Equation of motion:

	$\vec{F} = m\vec{a}$
$\hat{r}:$	$-\frac{G m_e M_m}{r^2} = -m_m r \frac{4\pi^2}{T^2}$

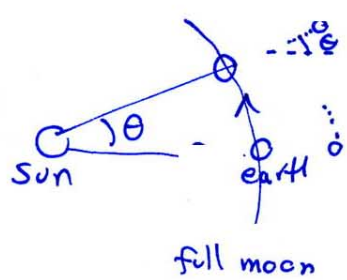
$$\Rightarrow \quad T^2 = \frac{4\pi^2 r^3}{G m_e}$$

Period:

$$\Rightarrow \quad T = 2\pi\left(\frac{r^3}{G m_e}\right)^{1/2}$$

$$= 2\pi\left(\frac{(3.8\times10^8\,m)^3}{(6.67\times10^{-11}\,N\cdot m^2/kg^2)(5.98\times10^{24}\,kg)}\right)^{1/2}$$

$$= 2.3\times10^6\,s = 27.0 \text{ days}$$



2nd full moon moon has moved an additional time equal to

$$\frac{1}{12}T = 2.25 \text{ days}$$

$$\Delta T_{full\,moon} = 29.25 \text{ days}$$

In-Class Problem 16: Consider a planet of mass m_1 in orbit around a extremely massive star of mass m_2. The period of the orbit is T. Assume that there is a uniform distribution of dust, of density ρ throughout the space surrounding the star and extending well beyond the planet with $\frac{4\pi^2}{T^2} > \frac{4}{3}G\pi\rho$. The gravitational effect of this dust cloud is to add an attractive centripetal force on the planet with magnitude

$$\vec{\mathbf{F}}_{dust} = -\frac{4}{3}G\pi\rho r m_1 \hat{\mathbf{r}}$$

in addition to the gravitational attraction between the star and the planet. You may neglect any drag forces due to collisions with the dust particles.

a) Briefly describe how you intend to model the motion of the object? What directions are you choosing for analyzing the components of your forces and state why?

b) Find an expression for the radius of the orbit of the planet.

c) If there were no dust present, would the radius of the circular orbit be greater, equal, or less than your result from part a). Briefly explain your reasoning.

Several billion years later, the dust cloud has vanished, but now assume that there is a repulsive force acting on the planet that is given by

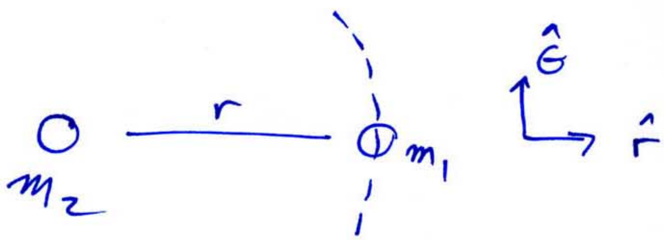
$$\vec{\mathbf{F}}_{repulsive} = \frac{k}{r^3}\hat{\mathbf{r}},$$

in addition to the gravitational force between the star and the planet The constant $k > 0$ and satisfies $G > \frac{2v}{m_2}\sqrt{k/m_1}$.

d) Show that there are two possible circular orbits for the planet that have the same velocity v. Find the radii of these orbits.

Solution: star with uniform dust cloud

circular orbit



dust cloud of density ρ, $m_{dust} = \rho \frac{4}{3}\pi r^3$

$$\vec{F}_{dust} = -G\frac{m_1 m_{dust}}{r^2}\hat{r} = -G\frac{m_1 \rho \frac{4}{3}\pi r^3}{r^2}\hat{r}$$

$$= \frac{-4}{3} G m_1 \rho \pi r \hat{r}$$

$$\vec{F}_{grav} = -\frac{G m_1 m_2}{r^2}\hat{r}$$

$\vec{F}$	$= m\vec{a}$
$-\frac{G m_1 m_2}{r^2} - G m_1 \rho \frac{4}{3}\pi r$	$= -m_1 r\left(\frac{2\pi}{T}\right)^2$

$$\Rightarrow \quad +\frac{G m_2}{r^2} + G\rho\frac{4}{3}\pi r = \frac{r 4\pi^2}{T^2}$$

$$\frac{G m_2}{r^2} = r\left(\frac{4\pi^2}{T^2} - G\rho\frac{4}{3}\pi\right)$$

$$\left(\frac{G m_2}{\left(\frac{4\pi^2}{T^2} - G\rho\frac{4}{3}\pi\right)}\right)^{1/3} = r_{dust}$$

note: $\frac{4\pi^2}{T^2} > G\rho\frac{4}{3}\pi$

with no dust present $\quad r_{no\,dust} = \left(\frac{G m_2}{4\pi^2/T^2}\right)^{1/3}$

$$r_{dust} > r_{no\,dust}$$

With repulsive force and gravitation:

$$\rightarrow \hat{r}$$

$$\vec{F}_{star,grav} \leftarrow \underset{m_1}{\bigcirc} \rightarrow F_{repulsive} \qquad -\frac{G m_1 m_2}{r^2} + \frac{k}{r^3} = -\frac{m_1 v^2}{r}$$

$$-G m_1 m_2 r + k = -m_1 v^2 r^2$$

$$r^2 - \frac{G m_2}{v^2} r + \frac{k}{m_1 v^2} = 0$$

$$r = \left(\frac{G m_2}{v^2} \pm \left(\frac{G^2 m_2^2}{v^4} - \frac{4k}{m_1 v^2}\right)^{1/2}\right) \Big/ 2.$$

note: $\frac{G m_2^2}{v^4} > \frac{4k}{m_1 v^2}$