

I will do another example, whereby you have to do an integral of some kind. I will take a thin sheet of matter, and let this be the dimension of that sheet: rectangular, with length a and length b . I will rotate it about this axis, which I call axis 1. It's going to rotate like this, or it may also lay back and forth. I'm interested in the moment of inertia of this sheet about this axis. It has mass m , and then I mentioned a and b . The mass is uniformly distributed.

I now have to add up all these mass elements-- m of i , and multiply them by their distances through the axis of rotation. Since this is a continuous mass distribution, I have to use an integral. I slice out here a ribbon, and the ribbon has width dx . The distance to the ribbon is x , so x equals 0 here at the rotation axis, and x equals a here at the edge of the sheet, the rectangular sheet.

So I-- about this axis of rotation now-- is going to be an integral from x equals 0 to a of this mass element times x squared, because that's the distance to this mass element. I first have to calculate now how much mass is in this ribbon. The mass that is in this ribbon is dx divided by a times m -- that is clearly dm . If now I want an integral, I take the whole sheet into account. I get dm , which is m over a times dx , but I also have to multiply by x squared, so I will put the x squared here, and I have the dx there.

This is not a very difficult integral: that integral becomes m over a times $\frac{1}{3} x$ to the third, and I evaluate that between 0 and a , because x goes from 0 to a . This becomes $\frac{1}{3} m$ times a squared. I lose one a here-- I get an a to the third and an a downstairs-- I have a squared.

This result, you can probably find in most tables. Most tables, not all, will give you the moment of inertia of this rectangular sheet if you rotated it about an axis that goes through one of the sides. This is not something you want to remember, but it's something that you see that you can derive.