I now want to calculate moments of inertia of objects with continuous mass distributions-- no longer discrete masses. When we have this discrete masses, then we can calculate the individual moment of the inertia of the individual masses, and add them all up. When you have a continuous mass distribution, you have to use an integral-- you have to integrate over the entire volume of the object, and that complicates matters a little bit.

I will start with a classic simple example of using a rod with uniform mass distribution. I have here a rod, and the mass is uniformly distributed-- the rod has length l it has mass m, and the mass is uniformly distributed. I'm going to rotate this rod exactly about its center, about an axis which is perpendicular to the rod itself. If this were that rod-- it's a pencil, but think of it as a rod-- then it would rotate about this vertical axis to the middle of the rod. It could oscillate, or it could rotate, and in both cases we would want to calculate the moment of inertia about that axis.

Now, we have continuous mass distribution, so the moment of inertia, as we defined it, is the sum of all the mass elements I times R i squared. Obviously, it has to be changed into an integral-- we have to integrate over this whole rod.

I slice out here a small section dx-- it's at the axis of rotation-- and this is at a distance x from the axis of rotation. I call this x equals 0, and x would equals minus 1/2 l here, and x equals plus 1/2 l here.

What is amount of mass that is in here, this little element dm, this mass dm? The fraction of the total mass that is in here is dx divided by l, the total mass is m, and this is the mass of this little section, which has length dx. If now I go to the continuous summation, which becomes an integral, then I get I-- I will put a c in there to remind you that it is an axis through the center-- which now becomes an integral over x, and the x goes from minus 1/2 l to plus 1/2 l. I get m over l, and then I get dx-- but I'll will wait with my dx, because I have to multiply with x squared-- and x is the distance from that mass elements to the axis of rotation. So I get x squared, which is this term dx.

This is a relatively easy integral: this is m over l times 1/3 times x to the third, evaluated between minus 1/2 l and plus 1/2 l. You will find it not difficult, I hope, to convince yourself that this is 1/12 ml squared. This is a rather well-known result: when you look up the tables of moments of inertia, you can almost find it any book what the moment of inertia is of a rod which is rotating about its center axis of rotation perpendicular to the rod and uniform mass distribution. If I were you, I wouldn't even try to remember this twelfth ml squared-- I certainly don't, but I want to show you that it can be derived in a relatively easy way.