

Let's first take a very simple example whereby I have two point masses which are connected through a mass less rod-- a little bit artificial, but later I will relax the situation. I have here a mass m_1 , I have here a mass m_2 , and I would assume that these masses themselves have no finite size-- it's a point mass-- so there are these two discrete masses. I rotate the system about an axis perpendicular to the line that connects them. Let this distance be r_1 , let this distance be r_2 , and I want to know the moment of inertia about this axis.

As we defined, moment of inertia is this sum of $m_i r_i^2$ over all the masses here-- in this case, i equals 1 to 2, because we only have 2-- r_i^2 , and r_i is the distance from the mass elements through the axis of rotation. That's relatively easy here. This is a distance from this mass to this axis, because it's already 90 degrees, and this is the distance from this mass to the axis. In this particular case, we find that I about this axis equals $m_1 r_1^2$ plus $m_2 r_2^2$.

Now imagine that I rotate this system about this axis-- again, perpendicular to the to the direction that connects the two masses. I want to know what the moment of inertia is about this axis. Since m_1 has no size, r_1 is now 0, there is no distance between this mass and this axis of rotation-- so the only mass that contributes to the moment of inertia is mass 2. We now find that the sum of $m_i r_i^2$ is simply $m_2 r_2^2$, the distance from m_2 through this axis, which is $r_1 + r_2$ squared.

This number is different from this number-- the two are clearly not the same, so this shows you that moments of inertia do depends very much on which axis you choose. It's not an intrinsic property of, in this case, two masses-- it depends on how you choose the axis. If I chose the axis to coincide with the mass less rod that connects the two, then the moment of inertia would be 0. If here is that point mass m_2 , here is the point mass m_1 , and here is the mass less rod that connects them, clearly if this is a point mass, the distance of this mass through this axis of rotation is 0, both for this one and for this one-- so here, you would have a moment of inertia which equals 0. There's a huge difference about how you choose the axis.

If I continue with this rather artificial situation whereby we have point masses-- m_2 , and we have a point mass m_1 -- point mass means no finite size, and this is a mass less bar that connects the two. If now I make them rotate about this axis at an angle θ -- and this is r_1 , and this is r_2 . I now have to remember that I equals the sum-- I equals 1 to 2 of $m_i r_i^2$, but r_i is the distance from the

mass to the axis of rotation. That distance is this: $r \sin \theta$, and this equals $r \sin \theta$ times the sine of theta. That distance here equals $r \sin \theta$ times the sine of theta.

Now the moment of inertia becomes $m_1 r_1^2 \sin^2 \theta + m_2 r_2^2 \sin^2 \theta$. I could write down that it is $\sin^2 \theta$ times $m_1 r_1^2 + m_2 r_2^2$. You can see immediately that when theta equals 90 degrees-- that means when the axis is perpendicular to the line that connects them, that you have a maximum. We already derived that, but when theta equals 0, and the axis is like this, then the moment of inertia equals 0. Here, the moment of inertia is the maximum value possible-- it very much depends on the geometry.