

I'd like to discuss now-- what is a moment of inertia? We will see moments of inertia come up several times during this course on Newtonian mechanics.

Whenever you want to know what the rotational kinetic energy is of an object-- some crazy shaped object rotating about an axis-- then the moment of inertia comes up. If the object has an angular velocity-- ω -- which could change in time. It could rotate continuously in one direction, or it could oscillate back and forth. If at any moment in time it would have a particular angular velocity-- ω -- then the kinetic energy of rotation equals $\frac{1}{2}$ times the moment of inertia of rotation about that axis times ω squared.

We will see it with kinetic energy. We will also see it when we derive oscillations. We will do problems whereby we oscillate a ruler-- it's a classic problem which I will do. I have a ruler here, and I have a hole. I have a pin here, and I put the pin through the hole-- it's almost frictionless, but not quite. I offset the ruler, and I let it oscillate.

When you want to calculate the period of oscillation, you will have to know the moment of inertia of rotation about this axis, and this moment of inertia will depend on where you stick the pin. The frequency of the oscillations, and therefore the period of the oscillation also depends on where you stick the pin. This is where moments of inertia come up.

How is the moment of inertia defined? Suppose we take a three dimensional object-- some crazy three dimensional object-- and think of it as being a potato, 3D. I rotate this potato about an axis. Here I have a potato, and-- maybe I can zoom in a little bit, and oh, I have to zoom in. So here's a potato-- oh, that's too far-- and I could stick in here a Phillips screwdriver. You can see that it could rotate about this axis. That is sort of this axis that I have here, but of course, I could put in the axis of rotation here-- I have options-- and then I could rotate it like so.

I have a lot of choices in terms of how I choose the axis of rotation. I could put it in here-- argh, it's not so easy-- and I can rotate it like this. The moment of inertia, as you will see, differs for different axes. Wow-- there's a lot of potato juice here. This is the axis of rotation that I have chosen here quite arbitrarily, and I carve out here a small mass element, which I call them m of i . I have to know the distance from that mass element m of i to the axis of rotation, so this is r of i .

I would have another one here-- say, some little mass element, and then I would have to know this value r of i . The moment of inertia is now defined: I , rotation, the moment of inertia about this axis is the sum all these values for m of i integrated over the entire potato-- the entire object-- times r of i squared, and r of i are the individual distances of these individual small mass elements through the axis of rotation.