

I now want to discuss the perpendicular axis theorem. The perpendicular axis theorem allows you to calculate a moment of inertia, but it only applies to thin sheets.

I place the thin sheet into plane  $xy$ . Here is my thin sheet-- you've seen it before in separate segments-- with width  $a$ , length  $b$ , uniform mass distribution, and mass  $m$ . I can choose any axis  $x$  and  $y$  in the plane through the sheet-- they are perpendicular to each other. What the perpendicular axis theorem tells me is that the moment of inertia about the  $z$  axis, which is perpendicular to the paper, is the moment of inertia about the  $x$ -axis-- which I can choose in this plane-- plus the moment of inertia about the  $y$ -axis, which I can choose also in this plane, but  $x$  and  $y$  are perpendicular to each other.

You can choose  $x$  and  $y$  in crazy ways, but I will not do that: I will choose the  $y$ -axis right through the center. I will also choose the  $x$ -axis right through the center  $c$ , right through the center of this sheet. I will call this  $x$ , I will call this one  $y$ , and  $z$  is the one that is perpendicular to the paper. I'm going to rotate it about an axis perpendicular to the paper-- I call this axis 1, and I call this axis 2.

$I$  of 1-- you can either look up in a table, or you derive it as we did in a different segment.  $I$  of 1 equals  $1/12$  times  $ma$  squared, and  $I$  of 2 about this rotational axis, equals  $1/12$   $mb$  squared. What this perpendicular axis theorem is was telling you that this is a thin sheet that the moment of inertia-- rotation about the  $z$ -axis perpendicular to the paper-- equals  $1/12$   $m$  times  $a$  squared plus  $b$  squared. If you're lucky, you may find this result in a table. If not, you may have to derive it as I just did.

Suppose we didn't want to know the moment of inertia about the axis  $c$ , as you see here, but suppose we wanted to know the moment of inertia about point  $P$  here, axis through  $P$  perpendicular to the paper, and that this separation between  $P$  and  $c$  is  $d$ . You guessed it: I already know the moment of inertia about  $c$ . If I want to know the moment of inertia about the axis point  $P$ , and these two axes are parallel, now I can apply the parallel axis theorem again. I know the vertical axis about point  $c$ , and so therefore the moment of inertia about point  $P$ -- a vertical axis perpendicular to the paper-- is now the moment of inertia about point  $c$  about this axis plus  $m$  times  $d$  squared. I will write that  $m$  divided by  $12$   $a$  squared plus  $b$  squared plus  $m$   $d$  squared.

I want you to remember this, because I'm going to use it later when I'm going to calculate the oscillation of a ruler with a pin through point  $P$ . It's going to oscillate back and forth in a gravitational field.

Remember this and keep this in mind: that's the moment of inertia about an axis through P of this uniform flat sheet which has width  $a$  and length  $b$ , and the separation to the center of mass is  $d$ . That's the separation.