Now let's go on to problem 8.35. In 8.35, we now deal with moments over inertia, which plays an equally important role in rotation, and you'll see the connection between the two.

I have to define an axis of rotation, and the moment of inertia about that axis of rotation is defined this way. This is scalar, it's not a vector, and it requires a little bit of an explanation.

Suppose I have a rod, and right through the center, I would have an axis, which I call a 1. The way that we defined r i is as follows: r i is measured from little mass elements m i perpendicular to the x's. That's the definition of this r i.

In this particular case, if I slice out here a little mass m of i, then I would have a position vector measured from this point, because that is perpendicular from m i to the axis. This would be r i, and if I sum this over all values of m i and all values of r i, then I will get the moment of inertia. There's never any that danger of it being a negative, because r i is being squared. If this were a rod with uniform length l, and the mass is distributed uniformly, then I 1-- 1 will indicate this axis-- equals 1/12 ml squared-- you can look it up in almost every table.

If you had a solid sphere-- so this is solid, and it is a sphere-- if the mass is uniformly distributed, and if I take an axis of rotation right through the center, if it has a radius r, if it has a mass m, and let's call this axis a 1, then I of 1 equals 2/5 m R squared. You don't want to remember these things-- you can look these up in tables, and you have some of the tables in your book.

There are two very key theorems that will help you to solve almost all moments of inertia, using the values that you see in the table and manipulating them a little bit-- one is called the parallel axis theorem. If I have an axis a 1 through the center of mass, and I have an axis a 2 which is parallel to a 1, but it is at a distance d, then I 2-- the moment of inertia ocean relative to do this axis-- equals I through the center of mass, which in this case, x is 1 plus md squared. It comes in extremely handy, and today, you're going to need it.

If we can take the case of the rod, the uniform rod, and we know what the moment of inertia I 1 is through the center of mass-- that was the 1/12 ml squared. If there is a rotation about another axis a 2 parallel to a 1, and the whole rod rotates about this axis, then I 2 equals I 1 plus md squared if m is the mass of the rod and this is d. This will come in very handy, and you can do the same with the spheres if you know the moment of inertia relative to this line. If you now have another axis with the distance d, then you can apply exactly the same equation.

Then there is another theorem, which is called the perpendicular axis theorem. This only works when you work with very thin sheets. Suppose you have a very thin circular disk-- very thin-- and I have an axis of rotation perpendicular to the paper. Let's call that the z direction. and so I z is this axis-- you may try to prove that, it's not so obvious-- the x-axis and the y-axis is also equal to I x plus I y. There is rotation about this axis of the plate, or rotation about this axis of the plate. You can also do that with this point here, so the moment of inertia rotation about this axis would be the same as the moment of inertia about this axis plus this axis as long as they are perpendicular to each other.

If we are very specific, the moment of inertia of a thin disk with uniform mass and this radius r, and has mass m, that is 1/2 mR squared-- this can be looked up in your tables. It follows immediately that I x equals 1/4 mR squared, because I x and I y must be the same for reasons of symmetry: they both go through this point, and there's no reason why one would be larger than the other, so this is 1/4 mR squared. If I pick any other axis through the center, and I call this a 2, then it is immediately also I 2.

Now, once you know I 2, you could easily find the moment of inertia of rotation about this axis, if the separation here is d, because-- let's give this axis a number, and let's call it a 3-- now the moment of inertia I 3 equals the moment of inertia going through the center of mass along the line a 2, so that is I 2 plus md squared. You see here that I have used both the parallel axis theorem, and I have used the perpendicular axis theorem, and sometimes you have to manipulate these things a little bit, and massage them a little bit, because you've stuck to the numbers in the specific cases given in your tables-- the tables in general are quite complete. You have disks, you have spheres, you have plates, cylinders, and rods-- but in any case, if your favorite one is now in there, you have to somehow manipulate it and massage it a little bit.

That's what's this problem is all about: you have a square plate. For now, I will call this b 2, and b 1-- although very shortly, I'll make them the same, because they are the same-- is a thin plate. You are being asked to calculate the moment of inertia of rotation about this axis, and let us call this axis a 5. This is at an angle of 45 degrees.

What I found in the study guides is that for the axis of rotation about this axis-- so the whole thing rotates about this axis, the flat plate-- that result, I 1, equals 1/3 m times b 2 squared. That's a given from your table. I will now forget the fact that b 1 and be 2 are different, because they're no, so we'll write down for this b, and we'll write down for this b, and so that is 1/3 m times b squared.

Now I have to somehow find this one. I'm first going to use the parallel axis theorem, and I'm going to use it to move this one to a parallel line right through the center of mass. Let's call this one a 2. It should be immediately obvious that I 2 plus m-- m is the mass of this plate-- times 1/2 b squared must be I 1. This is parallel axis theorem, because the separation d equals 1/2 b, so here you see your d squared.

Now we know what the moment of inertia is in rotation about this axis, when it rotates like this. I also know that if I take this axis perpendicular to the paper, and I call that a 3, then I know that I 2 must be 1/2 I 3, for the simple reason that this moment of inertia is the same as the sum of this one plus this one. It's immediately obvious that it is 1/2 I 3. I'm going to-- let me take a different color-- take an axis that goes through the center, but which is exactly parallel to this one, and I called this a 4. I want you to appreciate, even though that may not be so intuitive, that this I 2 must also be the same as I 4.

The reason being is that the moment of inertia about this axis-- the vertical axis-- must be the same as the moment of inertia about this axis plus the moment of inertia about this axis, which is perpendicular to it. You can call this x, and you can call this y-- so this one and this one obviously are the same because of symmetry. It's immediately obvious that this one, as well as this one, must be half of the moment of inertia about this axis, and so you see that I 4 becomes I 2. Now you're well on your way-- the ones you know with the moment of inertia is about a 4, and you would have no problems going from 4 to 5.

This our final goal-- you use, again, the parallel axis theorem, and you are on your way. You see in an example here whereby you have to manipulate things a little bit. It's almost like a puzzle, a crossword puzzle-- I like it, actually. It's kind of cute.