Let now get into cross products. Let C be the cross product of the vector A with the vector B. I'm telling you now that C will be perpendicular to A, and it will be perpendicular to B.

Suppose I have A here, I have B here, and let this angle between them be theta, so this my plane of the paper. It's through the plane of the vector A and B. The direction of the cross product must be perpendicular to my paper, then, because C is perpendicular to A, and C is perpendicular to B, so C has to be perpendicular to both A and B.

I have two choices now: perpendicular to this paper could be either out of the paper, or it could be into the paper. Now comes what I call the corkscrew rule. I always apply the corkscrew rule, and that is the following: you rotate A over the smallest angle to be, and if that's a counterclockwise rotation, then this vector, C, is coming towards me perpendicular to the paper. The vector is an arrow, and this is the tip of the arrow-- I see the tip, so it's coming towards me. If, however, I had B cross A, then I have to rotate B, because the one that is first mentioned over the smallest angle to A, that now is a clockwise notation, and now this vector goes into the paper.

How do you remember this corkscrew rule? I think that's very simple, and this is the way I learned it in high school. I always use it-- there are other ways to determine the direction of a cross product, of course, but I prefer this one. I have here a corkscrew, and I have here a cork. If I turn as seen from this side clockwise, then the screw goes into the cork.

I go this way, clockwise from this direction, and it goes in there. Notice when I go from B to A, I rotate my corkscrew clockwise, and the corkscrew goes in. Now, looking from this direction, I go counterclockwise. What just happened-- the corkscrew comes out again. Here I go from A to B counterclockwise, and so the corkscrew comes towards me.

I can do it with a potato, and then you may be able to see it even better. Maybe we can see it from above, even-- oh, this angle. I turn clockwise, and you can see that the vector goes in there. I turn counterclockwise, and the vector comes out-- I call this the corkscrew rule. You may not like this one, or you may have a different method to find the direction, and be my guest, but I always, in my head, apply the corkscrew rule.

It follows from what we just learned that A cross B equals minus B cross A, because remember, if this is coming vertically out of the paper, and this is going vertically into the paper, then they are 180 degrees apart. In a very different situation, we have with dot products whereby A dot B is the same as B dot A.

We know the direction of the cross product, because we discussed that in detail, and now comes the question: what is the magnitude of the cross products? If we have a vector A, and we have a vector B, then the magnitude of A cross B-- this indicates magnitude-- equals the magnitude of the vector A itself times the magnitude of the vector B itself times the sine of the angle theta. Look at this a little bit more closely--- if I make here a parallelogram, then I would claim that the magnitude of the cross product between A and B is this surface area. You can easily see that this is length A, but this length is B times the sine of theta-- this angle is theta, and this is also theta.

The area, B sine theta times A is this area, which is exactly the same as this area. It follows immediately that if theta is 0 degrees or 180 degrees, that the cross product is 0. The dot product is 0 when theta is 90 degrees, and then we have the dot product equal 0. When theta equals 0 degrees, or 180 degrees, then the cross product equals 0.

Now comes the $64 question: we know the direction of a cross product and we know the magnitude of the cross product. You may say, that's all very nice and dandy, but if you give me two vectors in three dimensions, how can I find the vector notation of the new vector with all three components, x, y, and z? I will give you an easy recipe for that, which is exactly consistent with what I told you, namely that we discussed the direction, and we discussed the magnitude. What I'm going to tell you now is not new, but it just a different representation of exactly the same thing. I will show you that the two are consistent.

If I have a vector A and a vector B-- and A comes first, and B comes second-- so C equals A cross B. Then I write down here some kind of a matrix: I write down x roof, y roof, z roof, A of x, A of y, A of z, B of x, B of y, and B of z. I repeat this here-- it looks like a bit of hocus pocus, doesn't it-- I repeat this here, and I repeat this here. Magic, isn't it? I repeat this here.

Be prepared now: now comes the easy recipe for you to remember the three components of the vector C, and this is what I always use. I start here at the x, and I go to the right hand corner. C of x-- that's the x component-- is the product of these two numbers, A y and B z minus the product of these components which is A z times B y. Now comes C of y-- that's the product of these two components plus A z times B x minus, and you guessed it, A x times B z. The z component would be A x B y minus A y B x.

The vector C-- of which this is x component, this is the y component, and this is the C component-- must be perpendicular to A, it must be perpendicular to B, and the magnitude of that vector must be the magnitude of A times the magnitude of B times the sine of the angle. It better be, because that is what I told you earlier is what a cross product is all about.

Let's look at that in some more detail. First of all, I can write down this vector C in a very elaborate way. You may not like that-- I don't-- but let's do it. C would then be A of y B z minus A of z B y x roof plus A of z B x minus A of x B z y roof plus A of x B y minus A of y B x z roof. That's the vector in all its beautiful glory.

Let's now take a vector A, and we take the one that we have worked with before: 3x roof minus 2y roof plus 4z roof. We take B, which is one we have also seen before: minus x roof plus 3y roof plus 2z roof.

What, now, is C? A y B z, and A y B z-- that's minus 4 minus A z B y. A z B y is 12 with a minus sign, minus 12, in the x direction, plus-- and you check the numbers that I'm going to write down now, you check them for yourself, and you better do that because I could make mistakes. I find that the vector C equals minus 16 x roof minus 10 y roof plus 7 z roof.

Now I want to check whether this vector, which is the cross product, meet all my previous conditions. It should be perpendicular to A, it should be perpendicular to B, and it should have the right magnitude. How do we know whether C is perpendicular to A? Now our knowledge of dot products comes up-- if we can demonstrate that the dot product between A and C is 0, their angle is 90 degrees. That's relatively easy-- dot products are not so very hard.

Let us see now whether indeed C dot A-- or A dot C, for that matter-- whether that is indeed 0. It's a requirement: they must be perpendicular to each other if I have done it right and if I have calculated C correctly. This equals C x A x plus C y A y plus C z A z. We know what A is, and we know what C is, so when I plug in the numbers, I find minus 48 plus 20 plus 28 equals 0. Yippee-- it is zero.

Now I can ask myself the question-- is C dot B also 0? It better be 0, because C has to be perpendicular to B, as well. When I worked that out, I find 16 minus 30 plus 14, and that is also 0-- yippee! I've shown you now that the vector that I found, A cross B, indeed is perpendicular to A, and is perpendicular to B.

Now comes the question: is the magnitude the right magnitude? Is the magnitude of C the magnitude of

A times the magnitude of B times the sine of theta? We know that the magnitude of C must be the square root of C of x squared plus C of y squared plus C of z squared, and that is easy to calculate. You know that C x is minus 16, C of y is minus 10, and C of z is 7. If you work this out, you will find that this is approximately 20.1-- it's the magnitude of the vector. What is this? A, you remember, was the square root of 29, B was the square root of 14, and theta-- as we calculated earlier for the combination A B-- was 92.8 degrees. If I substitute that in here, what do I find? 20.1, so I feel very happy that you see that the two are identical.

We found a vector in three dimensions using this recipe-- it's rather complicated. We were able to demonstrate that that new vector, that cross product C, is perpendicular to A, perpendicular to B, and it has the right magnitude. We did not check one thing-- whether it has the right direction. You still have the direction in this directions and the opposite direction. I will leave that up to you.

That's the one thing that I have not demonstrated yet, that it has the right direction. Remember-- the corkscrew will give you the right direction.