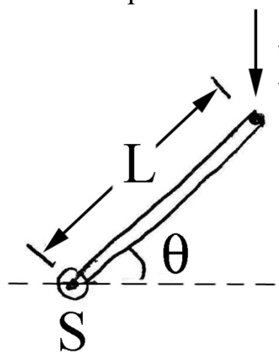
Rotational Dynamics
Concept Questions

Question 1: In the figure, a force of magnitude F is applied to one end of a lever of length L. What is the magnitude of the torque about the point S?



1. $FL\sin\theta$
2. $FL\cos\theta$
3. $FL\tan\theta$
4. None of the above

Answer 2. Choose units vectors such that $\hat{\mathbf{i}}\times\hat{\mathbf{j}}=\hat{\mathbf{k}}$, with $\hat{\mathbf{i}}$ pointing to the right and $\hat{\mathbf{j}}$ pointing up. The torque about the point S is given by $\vec{\tau}_s = \vec{r}_{sF}\times\vec{F}$, where $\vec{r}_{sF} = L\cos\theta\hat{\mathbf{i}} + L\sin\theta\hat{\mathbf{j}}$ and $\vec{F} = -F\hat{\mathbf{j}}$ then

$$\vec{\tau}_s = \left(L\cos\theta\hat{\mathbf{i}} + L\sin\theta\hat{\mathbf{j}}\right)\times -F\hat{\mathbf{j}} = -FL\cos\theta\hat{\mathbf{k}}\ .$$

So the magnitude of the torque about S is $|\vec{\tau}_s| = FL\cos\theta$. Note that the perpendicular moment arm is $r_\perp = L\cos\theta$, so the magnitude of the torque about S is $|\vec{\tau}_s| = r_\perp F$.

Question 2: Torque: Consider two vectors $\vec{\mathbf{r}}_{P,F} = x\hat{\mathbf{i}}$ with $x > 0$ and $\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} + F_z\hat{\mathbf{k}}$ with $F_x > 0$ and $F_z > 0$. The cross product $\vec{\mathbf{r}}_{P,F} \times \vec{\mathbf{F}}$ points in the

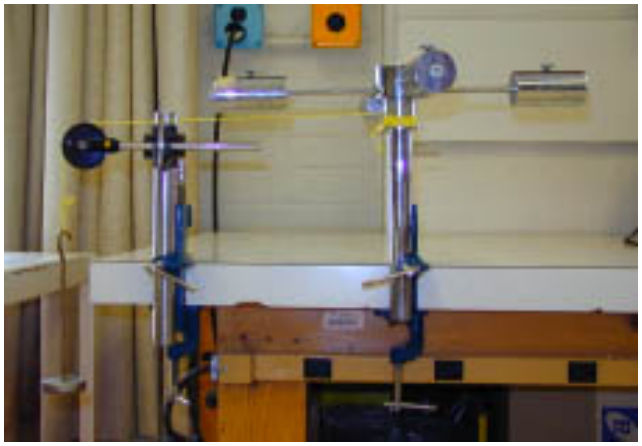
1) + x-direction
2) -x-direction
3) +y-direction
4) -y-direction
5) +z-direction
6) -z-direction
7) None of the above directions

Answer 4. We calculate the cross product noting that in a right handed choice of unit vectors, $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \vec{\mathbf{0}}$ and $\hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$,

$$\begin{aligned}\vec{\mathbf{r}}_{P,F} \times \vec{\mathbf{F}} &= x\hat{\mathbf{i}} \times \left(F_x\hat{\mathbf{i}} + F_z\hat{\mathbf{k}}\right) = \left(x\hat{\mathbf{i}} \times F_x\hat{\mathbf{i}}\right) + \left(x\hat{\mathbf{i}} \times F_z\hat{\mathbf{k}}\right) \\ &= -xF_z\,\hat{\mathbf{j}}\end{aligned}$$

Since $x > 0$ and $F_z > 0$, the direction of the cross product is in the $-y$ -direction.

Question 3: Chrome Inertial Wheel: A fixed torque is applied to the shaft of the chrome inertial wheel. If the four weights on the arms are slid out, the component of the angular acceleration along the shaft direction will



1. increase.
2. decrease.
3. remain the same.
4. Not enough information is given to decide.

Answer 2. torque about the central axis is proportional to the component of the angular acceleration along that axis. The proportionality constant is the moment of inertia about that axis. By pushing the weights out, the moment of inertia has increased. If the applied torque is constant then the component of the angular acceleration must decrease.