Two-Dimensional Rotational Dynamics
Challenge Problems

Problem 1: Estimation

Your car has a flat and you try to loosen the lugs on the wheel with a tire iron shown in the figure on the left below. You can't budge the lugs but then you try the 4-way wrench shown in the figure on the right and the lugs loosen. Based on the figures below, estimate how much torque you needed to apply to loosen the lugs.

Problem 2: Moment of Inertia and Rotational Kinematics: *Turntable*

A turntable is a uniform disc of mass $1.2\ \text{kg}$ and radius $1.3\times10^{-1}\ \text{m}$. The turntable is spinning at a constant rate of $f_0 = 0.5\ \text{Hz}$. The motor is turned off and the turntable slows to a stop in 8.0 s with constant angular deceleration.

a) What is the moment of inertia of the turntable?

b) What is the initial rotational kinetic energy?

c) What is the angular deceleration of the turntable while it is slowing down?

d) What is the total angle in radians that the turntable spins while slowing down?

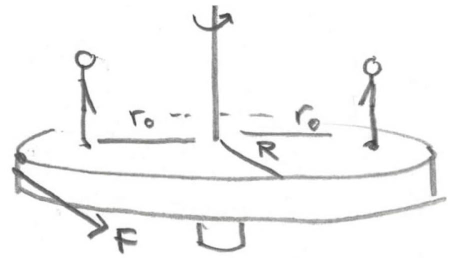
e) What is the magnitude of the frictional torque?

f) Find the work done by the frictional torque two different ways: use the work-kinetic energy theorem, and calculate the work done by the frictional torque through the total angle that the turntable moves while slowing down.

g) What is the average power of the frictional torque in stopping the turntable?

Problem 3:

A playground merry-go-round has a radius of $R = 4.0m$ and has a moment of inertia $I_{cm} = 7.0\times10^3 kg\cdot m^2$ about an axis passing through the center of mass. There is negligible friction about its vertical axis. Two children each of mass $m = 25kg$ are standing on opposite sides a distance $r_0 = 3.0m$ from the central axis. The merry-go-round is initially at rest. A person on the ground applies a constant tangential force of $F = 2.5\times10^2 N$ at the rim of the merry-go-round for a time $\Delta t = 1.0\times10^1 s$.



a) What is the magnitude of angular acceleration of the merry-go-round?

b) What is the angular speed of the merry-go-round when the person stopped applying the force?

c) What average power does the person put out while pushing the merry-go-round?

d) What is the rotational kinetic energy of the merry-go-round when the person stopped applying the force?

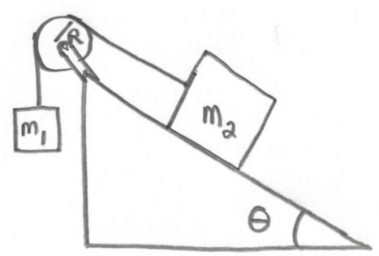
The two children then walk inward and stop a distance of $r_1 = 1.0m$ from the central axis of the merry-go-round.

e) What is the angular velocity of the merry-go-round when the children reach their final position?

f) What is the change in rotational kinetic energy of the merry-go-round when the children reached their final position?

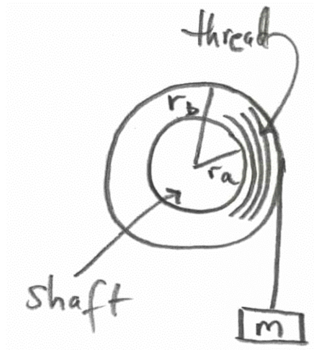
Problem 4:

A wheel in the shape of a uniform disk of radius R and mass m_p is mounted on a frictionless horizontal axis. The wheel has moment of inertia about the center of mass $I_{cm} = (1/2)m_pR^2$. A massless cord is wrapped around the wheel and one end of the cord is attached to an object of mass m_2 that can slide up or down a frictionless inclined plane. The other end of the cord is attached to a second object of mass m_1 that hangs over the edge of the inclined plane. The plane is inclined from the horizontal by an angle θ. Once the objects are released from rest, the cord moves without slipping around the disk. Calculate the speed of block 2 as a function of distance that it moves down the inclined plane using energy techniques. Assume there are no energy losses due to friction and that the rope does slip around the pulley



Problem 5: ***Stall Torque of Motor***

The following simple experiment can measure the stall torque of a motor. (See sketch.) A mass m is attached to one end of a thread. The other end of the thread is attached to the motor shaft so that when the motor turns, the thread will wind around the motor shaft. The motor shaft without thread has radius $r_0 = 1.2\times 10^{-3}\, m$. Assume the thread winds evenly effectively increasing the radius of the shaft. Eventually the motor will stall.



a) Suppose a mass $m = 5.0\times 10^{-2}\, kg$ stalls the motor when the wound thread has an outer radius of $r_f = 2.4\times 10^{-3}\, m$. Calculate the stall torque.

b) Suppose the motor has an unloaded full throttle angular frequency of $\omega_0 = 2\pi f_0 = 2\pi\left(6.0\times 10^{1}\, Hz\right)$ (unloaded means that the motor is not applying any torque). Suppose the relation between angular frequency ω and the applied torque τ of the motor is given by the relation

$$\omega = \omega_0 - b\tau$$

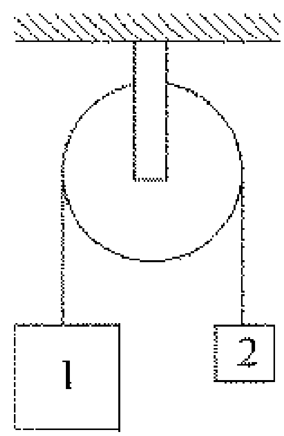
where b is a constant with units $s/kg-m^2$. Using your result from part a), calculate the constant b. Make a graph of ω vs. τ.

c) Graph the power output of the motor vs. angular frequency ω. At what angular frequency is the power maximum? What is the power output at that maximum? Briefly explain the shape of your graph. In particular, explain the power output at the extremes $\tau = 0$ and $\tau = \tau_{stall}$.

d) What torque does the motor put out at its maximum power output?

Problem 6: Torque and Angular Acceleration: ***Atwood Machine Solutions***

A pulley of mass m_p, radius R, and moment of inertia about the center of mass I_{cm}, is suspended from a ceiling. An inextensible string of negligible mass is wrapped around the pulley and attached on one end to an object of mass m_1 and on the other end to an object of mass with $m_1 > m_2$. At time $t = 0$, the objects are released from rest. How long does it take the objects to move a distance d? If $I_{\text{cm}} = \frac{1}{2} m_p R^2$, what does your answer reduce to?



We will use Newton's Second Law, and the Rotational Law to find the acceleration of all the objects in the system. We can then use kinematics to calculate the time it takes for each object to move a certain distance. As a strategy we will answer the following questions.

a) What are the free body force diagrams on the pulley and the two objects. Is the tension in the string constant?

b) Write down Newton's Second Law for the pulley and the two objects.

c) Write down the rotational equation of motion for the pulley.

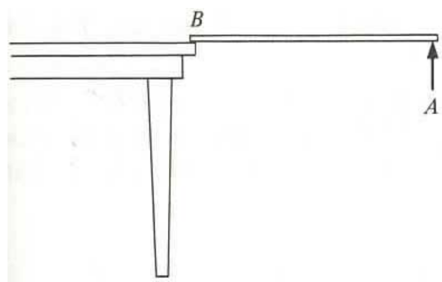
d) What is the constraint condition between the magnitude of the angular acceleration of the pulley and the magnitude of the acceleration of either object.

e) Find the direction and magnitude of the acceleration of the two objects.

f) How long does it take the objects to move a distance d? If $I_{\text{cm}} = \frac{1}{2} m_p R^2$, what does your answer reduce to?

Problem 7:

A uniform stick of mass m and length l is suspended horizontally with end B at the edge of a table and the other end A is held by hand. Point A is suddenly released. At the instant after release:



a) What is the torque about the end B on the table?

b) What is the angular acceleration about the end B on the table?

c) What is the vertical acceleration of the center of mass?

d) What is the vertical component of the hinge force at B? Does the hinge force have a horizontal component at the instant after release?