Simple and Physical Pendulums
Challenge Problems

Problem 1: ***Pendulum***

A simple pendulum consists of a massless string of length l and a pointlike object of mass m attached to one end. Suppose the string is fixed at the other end and is initially pulled out at a small angle θ_0 from the vertical and released from rest. You may assume the small-angle approximation, $\sin\theta_0 \simeq \theta_0$.

a) How long (period) will the pendulum take to return to its initial position?

b) What is the angular frequency of oscillation?

c) What is the speed of the mass at the bottom of its swing?

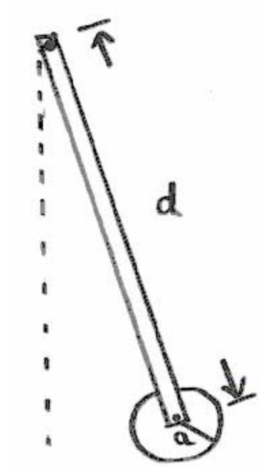
d) What is the angular velocity of the mass at the bottom of its swing?

e) Is the angular velocity the same as the angular frequency for the pendulum? Why or why not?

f) Why or why not does the period depend on the mass of the object?

Problem 2: Physical Pendulum

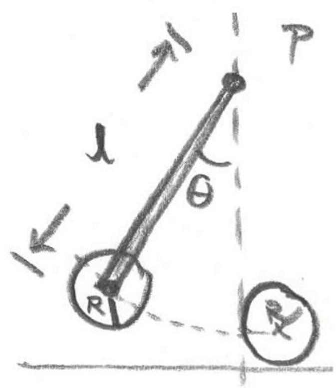
A physical pendulum consists of two pieces: a uniform rod of length d and mass m pivoted at one end, and a disk of radius a, mass m_1, fixed to the other end. The pendulum is initially displaced to one side by a small angle θ_0 and released from rest. You can then approximate $\sin\theta \cong \theta$ (with θ measured in radians).



a) Find the period of the pendulum.

b) Suppose the disk is now mounted to the rod by a frictionless bearing so that is perfectly free to spin. Find the new period of the pendulum.

Problem 3: A physical pendulum consists of a disc of radius R and mass m_1 fixed at the end of a massless rod. The other end of the rod is pivoted about a point P. The distance from the pivot point to the center of mass of the bob is l. Initially the bob is released from rest from a small angle θ_0 with respect to the vertical. At the bottom of the bob's trajectory, it collides completely inelastically with another less massive disc of radius R and mass m_2, $m_1 > m_2$.



a) What is the period of the bob before the collision?

b) What is the velocity of the bob just before the collision at the bottom of the bob's trajectory?

c) What is the velocity of the bob and disc immediately after the collision?

d) What is the new period of the pendulum after the collision?

e) What angle does the pendulum rise to when it next comes to rest?

Problem 4:

A wrench of mass m is pivoted a distance l_{cm} from its center of mass and allowed to swing as a physical pendulum. The period for small-angle-oscillations is T.

a) What is the moment of inertia of the wrench about an axis through the pivot?

b) If the wrench is initially displaced by an angle θ_0 from its equilibrium position, what is the angular speed of the wrench as it passes through the equilibrium position?