

A requirement for static equilibrium is that the sum of all forces on an object be 0, but also the sum of all torques relative to any point that you choose be 0. Those two are sufficient and necessary requirements. Let's write this down: the requirement for static equilibrium is that the sum of all forces on that object will be 0, and that the sum of all torques relative to any point that you choose also be 0.

Let's do a simple example. You have a bar resting on the floor-- there is friction-- and the bar is held up with a string, a mass less string. Here is that bar-- there is uniform mass distribution-- the mass of the bar is capital M, the length of the bar is L, and it makes an angle theta with the horizontal table. This bar is attached to a mass less string, and to make our life little more simple, we'll make this angle 90 degrees. This is the center of the bar, C, then let's call the end of the bar B, and here, where it rests on the floor, we call that A.

Through the center of mass, we have the gravitational force Mg. Right here, we have the tension in this mass less string-- let this be the tension. At A we have a contact force from the floor acting on the bar, something in this direction. I will decompose that two directions, one parallel to the floor, and one perpendicular to the to the floor. The one perpendicular to the floor we call M-- this is the normal-- and the one parallel to the floor, we call that the frictional force.

The frictional force must be in this direction as you can immediately see. Imagine when you hold this object like this on a string, it would want to slide in this direction, so the frictional force must be pointing in this direction. The frictional force could be way lower than the maximum allowed frictional force. It will be just so large that this object is not sliding and that this object is in static equilibrium.

Let's call this the positive y direction, and this the negative y direction. We'll call this the positive x direction, and this the negative x direction. I will decompose the tension into a force in the y direction, which is T cosine theta, and into one in the x direction, which is T sine theta. These two black forces replace this one.

The sum of all forces has to be 0. First, we take the x direction-- F of f is in the positive direction, and T sine theta in the minus direction. This equals 0, and I call that equation one. The sum of all forces in the y direction have to be 0-- plus T cosine theta plus N minus Mg equals 0. That's equation number two.

How many unknowns do I have? I have three unknowns: I don't know N, I don't know T, and I don't

know the frictional force. I do know θ , I know capital M , and I know the gravitational acceleration g . Two equations with three unknowns cannot be solved, but we do have one other requirement for static equilibrium-- we haven't used that yet-- that the sum of all torques relative to any point also has to be 0.

Let's choose point A. What is the torque relative to point A? N and a frictional force have no effect because they go through point A, so I have only this force, and I have the tension. Let us adopt a convention: I will call a torque that is for perpendicular to the paper and coming out of the paper upwards minus, and I will call the torque that goes into the paper, down into the paper, perpendicular to the paper, I will call that plus. If you look here, then you'll see that the torque relative to point A due to this force is going to be positive, and this one is going to be negative.

For this torque in the paper, I use the corkscrew rule-- this torque goes in the paper, and this torque counterclockwise comes out of the paper. I have to know the magnitude Mg of this force, multiplied by the distance from point A to that force. This distance equals $\frac{1}{2} L \cos \theta$ if the length of the bar is L , and I get a torque relative to point A that equals 0-- that is plus $Mg \frac{1}{2} L \cos \theta$. The torque relative to point A to the tension T is negative, and since I chose this angle 90 degrees, it's simply the force times the length L of this bar, so this is going to be minus T times L .

Notice that my L vanishes, and so this is my third equation. No unknowns were added, and so I now have three equations with three unknowns-- one, two, three equations, and the unknowns are N , normal force, T , the tension, and the frictional force F_f . Now I can solve for all three. Had I chosen point B, or any other point for that matter about which I would have evaluated the torque, then I would have obtained a different equation. However, that equation would not give me any new information.

Let's calculate the torque relative to point B. That torque must also be 0, so now the tension no effect because it goes through B. We have a torque due to this force N , which is going to be a positive one-- it's the force multiplied by the distance. to that force, and the distance to that force equals $L \cos \theta$, so we have N times L times $\cos \theta$.

Then we have this force, which is a negative torque because it's coming out of the paper, so it's Mg times the distance from point B to that force. This distance equals $\frac{1}{2} L \cos \theta$. I get minus Mg times $\frac{1}{2} L \cos \theta$. I also have a negative torque due to the frictional force-- it is the force times the distance from B to that force, and that distance equals $L \sin \theta$. I guess here minus F_f friction $L \sin \theta$, I lose my L , and this could then be my equation number four.

I leave it up to you to show that equation one, two, and three contain all the information that you need, and that number four doesn't add anything. Number four is a linear combination between one, two, and three. You have a choice: you could pick one, two, and three, and calculate your unknowns, or you could pick one, two, and four and calculate your unknowns. If you use all four, that doesn't give you any advantage.

If you had preferred to calculate the torque relative to point C, and then get 0, you could have done that. You could even have chosen this point here-- a ridiculous choice, by the way-- somewhere outside the bar, and you could have said that the torque relative to that point D was 0. That would have given you equation five.

Whatever you want to do, you always have to choose three equations, and you have to make sure that the three that you pick are not linearly dependent. If you pick one and two, with one torque equation, then you can't go wrong.