

This is problem 6.10.

Here we have a rather unphysical two mass less pulleys-- this is a pulley, which is mass less, and here there is a pulley which is mass less. There is a string over the pulley, and the string is also mass less-- it goes under here, and goes through there. This is a string, the string is mass less, the pulleys are mass less, it's attached here, the pulley is hanging here, and there's also no friction anywhere here, so it's a little bit artificial. This has a weight-- a mass, hanging capital  $M$ , and on this pulley is hanging a weight little  $m$ . The angle here is  $\theta$ , and the angle here, which happens to be the same, is also  $\theta$ .

We're being told that there is equilibrium-- in other words, that  $m$  is not going down, and capital  $M$  is not going up. That is only the case if the sum of all forces is 0. I will decompose [UNINTELLIGIBLE]  $y$  and  $x$  direction-- it's often very handy. It means that the sum of all forces in the  $x$  direction must be 0, and independently, the sum of all forces in the  $y$  direction must be 0.

Let's make free body diagrams-- here, it's easy. We have  $mg$ , and here we have tension  $T$ . The two must be equal-- otherwise, there would be acceleration. If the tension is  $T$  here, then the tension must be the same everywhere in the rope, because the rope is mass less. Imagine that I had a little section of rope here, and that the tension here would be different from the tension here, then this mass less rope would get an acceleration which is infinitely large-- zero mass, the two tensions would be different, get infinitely high acceleration, so that is clearly not allowed. That's nonphysical.

Therefore, whenever you deal with a mass less string, and no friction here, and no friction there, the tension along the string is everywhere the same. I can also put a tension here, because that is important for the force acting on this pulley. I could have a tension down here, and I could have put a tension down here, but we're not dealing with that here, so I left them out. Here I have  $mg$ .

On this object there three forces: these two tensions plus  $mg$  on this all object, capital the tension plus capital  $Mg$ . Some of the forces in the  $x$  direction are immediately 0, and I don't have to demonstrate that to you, because these are the two horizontal components of  $T$ . It's clear that they immediately cancel each other out.

Now in the  $y$  direction: in the  $y$  direction,  $T$  must be equal to  $Mg$  if we have equilibrium. That's one, and for this one, the vertical component of these two tensions must exactly cancel out  $Mg$ . This angle is

theta, this is also theta, and so this vertical component is  $T \sin \theta$ , but I have two of them-- so what do I get? I get to  $2T \sin \theta = mg$ . That is my second equation.

Now suppose theta and little m are known-- then I can find capital m, because I can't eliminate T from these two equations. You will find-- which is a rather trivial-- that capital M is little m divided by 2 times the sine of theta. For a given value of little m, and for a given value of theta, you immediately find what capital M is. If you were interested in the tension, you can either use equation one, or you can use equation two, and you would be able to solve for the tension.

We're going to pull M down-- we're going to pull this one down-- and we want to know what's going to happen. I will tell you what's going to happen-- the system is going to oscillate, but I want to be a little bit more quantitative about that. If I pull one down, then I pull down with a force which I call force Walter Lewin, and the only way that there can still be equilibrium is that the tension on capital M must go up while I'm holding it in my hands.

Here is that object capital M. I have here  $Mg$ , I have here the force by Walter Lewin, and this T-- which I will call T prime, as not to confuse you with the previous one-- must exactly balance out these two forces. To have equilibrium, I would get  $T' = Mg + F_{\text{Walter Lewin}}$ .

For the other object, for little m, I would get that  $2T' \sin \theta' = mg$ , because the tension is the same everywhere, times the sign of theta prime-- theta has changed-- equals mg. This is equation number three, and this is equation number four. As I pull capital M down, it should be obvious that little m goes up.

I can't find my drawing anymore, but-- I can't find my drawing anymore. It must have somehow disappeared. Oh, it slipped off all the way-- well, it's all the way there on the floor, so we'll just leave it on the floor.

It's immediately clear that if I pull this one down, that little m will go up, and that the angle theta will become smaller. Let us assume that I let go all of a sudden, so this force all of a sudden becomes 0, but the angle theta prime is not changing instantaneously. Immediately, T prime goes down-- so this T prime goes down-- and as this T prime goes down, notice that there is no longer equilibrium here. You now have a net force here which is not zero.

The net force on this little m, on this little object m, is now in the-- if I pull this down, and this object went

up, the net force is in the down direction.  $mg - 2T'$  and I'll call it now double prime, which is now the tension immediately after Walter Lewin lets go-- times the sine of theta prime equals  $m$  times the acceleration in the  $y$  direction.

This is no longer 0-- remember, this was 0 here when there was equilibrium. When  $T'$  goes down, and theta prime remains constant, this becomes a positive value, and so the object is being accelerated downwards. It's immediately obvious that if I had pushed little  $m$ , then capital  $M$  would have gone up, and that the whole system would've started to oscillate in this direction, and so you get an oscillatory situation.

It may be a challenge for you at any moment in time for a given value for theta to calculate what the tension  $T$  is-- I call it here  $T''$ -- as a function of the acceleration in the  $y$  direction. I will not do that now.

I mentioned to you that  $T'$  goes down at the moment that I let go. Don't think that  $T'$  becomes the same value as  $T$ , as we had before, when Walter Lewin was not touching them. No--  $T'$  will indeed go down to  $T''$ , but will not become as small as  $T$ . It will be larger, and you can see that it has to be a little larger, because otherwise this object would not accelerate upwards. If I pull it down, and I let it go, it's being accelerated upwards.

Give that some thought, and if you have some time, work out the equations while the motion is underway.