

## MITOCW | MIT8\_01SCF10mod06\_02\_300k

There are many problems I have selected, as always a few, in this case, 7 that I think will help you the most. You ready? I am.

There's an astronaut-- problem 3.2-- and the astronaut is on the moon. And the astronaut is throwing up an object at a certain angle  $\theta$ , and the object reaches the height  $h$ . It reaches the highest point at a time  $t$  of  $h$ . That is my shorthand notation. This is the  $x$  direction. And let this be the  $y$  direction. There is an acceleration,  $g$ , and I don't even know what it is. I didn't even look it up. We don't need it. But what is important that whenever you have an acceleration in the  $y$  direction, you better write down for it minus  $g$ .

The maximum distance that it travels in the  $x$  direction, that's called its  $x$  max. And the time that it hits here as we have seen last time is  $2t$  of  $h$  because of the complete symmetry of the parabola. There is a velocity with which the astronaut throws up the stone. And as I said, she does it at a certain angle  $\theta$  and the velocity in  $x$  direction, which never changes, equals  $v_0 \cos \theta$ .  $v$  doesn't change because there's no acceleration in this direction. The velocity in the  $y$  direction, which starts off at  $v_0 \sin \theta$  does change. It comes to a halt here,  $0$ , and then it increases and it reverses direction.

Now the idea is that this astronaut is going to throw up this stone with the same angle  $\theta$ , but with twice the velocity  $v_0$ . And all you're being asked, does the flight last maybe twice as long or maybe four times as long? Is the height that it reaches twice as high or maybe four times as high? And does it travel maybe four times as far or maybe twice as far? And those are issues that we will discuss now.

I will separate  $x$  and  $y$  as I have always done. And then you will see that this problem is not all that tough. In the  $x$  direction there's no acceleration, so the velocity remains constant. So  $v$  of  $x$  is any moment of time  $t$  equals  $v_0 \cos \theta$ . That is my equation number 1 and  $a$  of  $x$  equals  $0$ . The position in the  $x$  direction as any function any moment in time equals  $x_0$ , which I will conveniently choose  $0$ . So I choose this origin as  $0$  both for  $y$  and for  $x$ . Plus  $v_0 \cos \theta$  in the  $x$  direction times  $t$ . But the  $v_0 \cos \theta$  at times  $t$  equals  $0$  is the  $v$  at any time in the  $x$  direction. So that's equation number 2.

Now in the  $y$  direction, I have that  $y$  as a function of time would be  $y_0$ , which I also choose  $0$ . Plus  $v_0 \sin \theta$  times  $t$  minus  $\frac{1}{2} g t^2$ . That's equation number 3.

And the velocity in the  $y$  direction as a function of time equals the velocity in the  $y$  direction at time  $t$  equals  $0$  minus  $g t$ . And that is my equation number 4.

So let's first ask the question, when is  $y$  equal  $h$ ? In other words, when is  $v$  of  $y$  equals  $0$ ?

So I go to equation number 4. I substitute in here  $v$  of  $y$  equals  $0$  and I find immediately that the time that  $h$  is reached, which we have called  $t$  of  $h$ , equals  $v_0$  times the sine of  $\theta$  divided by  $g$ . That's the result that I will frame in red if you can see colors. The time that it takes to reach the highest point. So it should be immediately obvious since the time of flight, the whole trajectory, is  $2 t$  of  $h$  is therefore  $2$  times  $v_0$  times the sine of  $\theta$  divided by  $g$ . That you can now immediately answer the question that if  $v_0$  doubles, but if  $\theta$  does not change that the flights last twice as long. It's as simple as that. Because the duration of the flight is proportional to  $v_0$ .

Now how high does the object go if we double  $v_0$ ? Well, I substitute in equation 3, I substitute  $t$  equals  $t$  of  $h$ . Because that's the time that it reaches the highest point. And when I do that-- and you can do that for yourself-- I find that the height that it reaches equals  $1/2 v_0$  squared times the sine squared of  $\theta$  divided by  $g$ . And that is proportional to  $v_0$  squared. Let's also put this in a nice red frame. And so you see here that when you double  $v_0$ , all things being equal and  $g$  on the moon is the same all the time, so you don't have to worry about  $g$ , notice that when you double  $v_0$ , that it goes four times higher.

Then there is one question left. And that's the one I would like you to deal with. And that is the distance traveled. That is the horizontal distance traveled. And I think you should do that. So when we double  $v_0$ , how much further does it land in the  $x$  direction when  $v_0$  is doubled?

OK, I don't think this one's too tough, and so let's go on to our next problem.