So now let's go to 1.44, which is a classic application of our famous equations 1 and 2. 1.44.

Someone kicks a soccer ball and it's kicked at a certain angle. And the soccer ball reaches a certain height and the soccer ball returns to the ground. This is y, and this is x. I call the position where the ball hits the ground, I call that x hit. This height is h. The highest point of the parabola, so this is h. I call this position x of h to remind you that that's the moment it is at h. And this is the position x and y equals 0.

I'm going to calculate the-- answer the questions by completely separating x from y. There is an acceleration or you can call it a deceleration, it's a matter of words, only in the y direction. The velocity in the x direction remains constant. But there is an acceleration in the y direction, which we call g since we deal with gravity, and that is 9.8 meters per second square.

The acceleration is a vector itself is in this direction. And therefore, in all our equations where we will need the acceleration a in the y direction, we will write down minus g. And if we have to put in a number, it will become minus 9.8.

At time t equals 0, this is t equals 0. The object has a certain velocity. That velocity equals v0. And I will decompose that into velocity in the x direction, which is v0 cosine theta if this angle is theta. And it has a component in the y direction, which is v0 sine theta.

This one is high in the beginning, it becomes 0 here when the object is at its highest point. So right here there is no velocity in the y direction. And then as it comes back, the velocity increases again and it reaches a [? maximum ?] when it hits the ground. In the x direction the velocity remains constant all the time. There's no acceleration in the x direction.

So let's now apply our equations doing it first in the x direction, a of x equals 0. So we get that the velocity in the x direction at any moment in time equals v0 cosine theta. Let me call that equation 1. The position x as a function of t of this soccer ball will be x0, which for definition I will call 0 here. I can choose that. Plus v0 cosine theta times t. I will call that equation 2.

In the y direction where a of y equals minus g, I have that the velocity in the y direction at any moment in time starts off at v0 sine theta and then I get minus dt. Let's call that equation number 3. And the position where this in the y direction equals y0, which I choose conveniently 0 plus v0 sine theta. That's the velocity in the y direction at time t equals 0. Times t minus 182 g t squared. So that is my equation number 4.

Now the first question is where does the ball hit the ground? When the ball hits the ground I would say I use equation number 4. y equals 0 when the ball hits the ground. If I substitute y equals 0 in this equation, then I find two solutions for the time. I find t equals 0 and I find a second time when it hits the ground when it is at the ground, which is t hit, and that then becomes 2 v0 times the sine of theta divided by g. So this is when it starts, when it is at y equal 0 and when it hits the ground again, y is again 0. And we find this immediately from equation number 4.

So where does it hit the ground? Well x hits then equals the velocity in the x direction, which doesn't change. It's v0 cosine theta times the time that it has traveled. And this is the time that it has traveled to hit the ground. So I get a 2 here and I get a square here, and I get a sine theta here. And I divide by g. And this can be simplified a little bit. z0 squared times the sine of 2 theta divided by g. Because 2 times the sine of theta times the cosine theta equals the sine of 2 theta.

If you look at this equation, you can immediately see that for a given value for z0, the maximum possible distance that an object can travel will be if you kick it at an angle of 45 degrees. Because that you will get the when the sine of 2 theta is a maximum that is 1. And that's the case when theta equals 45 degrees. So you can never hit it any farther for a given value of v0.

Let's now calculate what h is. What is so special about h? Well, the y velocity when the object reaches its highest point equals 0. So I take equation 3 and I substitute in there that the velocity equals 0. And what do I find? That t of h-- that's the moment that it reaches the highest point-- equals v0 sine theta divided by g. And this is 1/2 of t hit. And that is no surprise. I want to remind you here what t hit is. You see you see the same value as the 2 here. You don't have the 2 here. And the reason why this is 1/2 t hit is immediately obvious. This is a parabola. The whole problem is completely symmetric about this moment in time. So the time that it takes for the object to reach the highest point is the same time that it takes for the object to go back to earth and hit the ground here, and so it is no surprise that it takes 1/2 the time to reach this point than it takes to come back to the ground again. And that of course, all I've said if I ignore all forms of friction, so there's absolutely no air drag whatsoever.

What is this height h now? Well I go back to equation number 4 and I substitute in that equation t equals t h and I have calculated here t h. So I find then that h equals v0 squared sine squared theta divided by g. That is the first term of equation 4 minus 1/2 a t squared. Here is my a and here comes my t squared. v0 squared sine squared theta divided by g squared. They each [INAUDIBLE] up. This term is exactly the same as this term. This is 1/2 and this is a 1. So h equals 1/2 times v0 squared times sine squared theta divided by g. And that's what you were asked to calculate.

I want to look at this equation in a little bit more detail to see whether my stomach-- to see whether my intuition agrees with the location of the three variables: v0, theta, and g.

Let us in our minds increase the value or v0. So we don't change theta. I just shoot it up at a higher velocity. It's immediately obvious that it will get higher. I get a parabola, which looks like this. So it's very pleasing that if v0 goes up that h goes up. It's not so obvious that h goes up with the square of v0. But at least you want the v0 upstairs.

When theta is larger, if you freeze v0, so v0 is a constant, but you make theta larger, then the y component of the velocity increases. And in fact, reaches a maximum value when theta is 90 degrees. It's immediately obvious is the y component of the velocity is larger that the object will go higher. So if I make theta larger, you would get something like this. Goes higher, doesn't reach so far either.

So when theta is larger, it's clear that h should go up. When g is larger, I claim it's obvious that h is lower.

Imagine that g were 0. There wouldn't be any deceleration or acceleration in the y direction, and the object would continue to go along a straight line. Here [INAUDIBLE].

So if g were 0, if this were somewhere in outer space and there were no gravity, the object would go along a straight line. So this is 0, h would be infinitely high. If g is huge, if g is infinitely large, the soccer ball wouldn't even get off the ground. So you see, it's nice that g is below the line, even though it's not obvious why you have the square or sine theta and why you have the square of v0 and why you have g [INAUDIBLE]. But at least your locations are just perfect. My stomach is happy. My intuition was correct.

OK, now we go on to revisit this x hit. I want to say a few more things about x hit. x hit was v0 squared times the sine of 2 theta divided by g. Notice if I specify x hit, there are a zillion combinations of theta and v0 for which I can satisfy this equation. If I choose x hit and if I also choose a value for [INAUDIBLE] v0, so I freeze x hit, I specify what this should be, and I specify with v0 should be, then there are two angles theta, which satisfy this equation. And that may not be so obvious. There are two solutions to that equation.

The reason is that the sine of 2 theta is the same as the sine of 180 degrees minus 2 theta. That's simply trigonometry. And therefore, if you throw the soccer ball up at angle theta or if you throw the ball up at an angle 90 degrees minus theta, you get exactly the same answer for this equation. So 20 degrees and 70 degrees give you exactly the same distance. 10 degrees and 80 degrees will give you the same distance. 30 degrees and 60 degrees will give you the same distance.

Here, this goes over 30 degree. Bingo. This goes off at 60 degrees. Bingo. You get two solutions. They have very different values of h. They have the same value though of v0. This v0 is the same as this v0, but the angle theta is different in the two cases.

So this is a good moment, I think, to work on the problem with the strobed ball.

We have a car and the car is moving with a constant speed. You see this wonderful, very exciting demonstration in lectures. I remember I saw it when I was a kid. I think I was 9 years old when I saw it in a museum of science. And I was intrigued by it.

So there's a train moving at a constant speed. There's a ball. The ball is shot up vertically seen from the train and when they come together, the ball hits the train again. And so if I make a drawing of that, this is the trajectory of the train and the train is at these locations, which are 1/10 of a second apart. The strobing occurs at time separation of 1/10 of a second. So the ball is on the train here. The ball is on the train here. Somewhere here the ball is shot up vertically as scene from the train. Not as seen from the seats in 26/100. What you see from your seats in 26/100 is this. And so you would see your strobe picture, you see a ball here, you see a ball here, here, here, oh, I'm lucky. Almost at the highest point. That's lucky of course. That's not always the case that you hit the highest point when you strobe it. Here, here, and here. Here, not here. Because I strobe at this moment and then I see the ball here again. This is the picture that you will see.

You also have a calibration. You know that this is 1 meter in the y direction. You know that this is 1 meter in the x direction. So you can calculate at any moment in time special distances here and special distances in this direction.

The first question is you're being asked to calculate what the velocity is of the car. The car has only a velocity in the x direction. Now the way I would do that, I would measure this distance. Take my ruler, I call this delta x, and I'm going to count 1, 2, 3, 4, 5, 6, 7, 8 9, 10, 11. It takes 11 flashes, exactly 11 flashes, so the velocity of the car equals this distance delta x, which you have measured with your ruler. You calibrate it against this one divided by 11 flashes times 0.1 seconds. So you know the velocity of the car.

Now you may ask, why do I do the whole base? Why don't I go for just the difference in x position between two flashes? So why don't I measure this distance here, put that in here, and divide that by 0.1? The reason is very simple. That if I measure this distance, there's always an uncertainty in my measurements, which might be 1 millimeter in my ruler. I may not be able to tell this distance any better than 1 millimeter. And then this distance I could probably also tell no better than an accuracy of 1 millimeter. But 1 millimeter out of this distance is a percentage error, which is smaller than the percentage error if I take 1 millimeter out of that small portion. And since percentage errors is all that counts in physics, we call them relative errors, my relative error would be the smallest if I take this largest base, and that's why I will take all the way from here to here.

So I know what the velocity is now of the car. And now you're being asked from this strobe picture to calculate g. What is g?

Well you may think that if you can calculate what the time is when it hits the highest point here, and you may think that if you measure h itself, which of course can easily be measured with your ruler, you may think that you can calculate g. But that is not so.

Somehow you must know what the velocity is at time equals 0 when the ball leaves the gun. You must know what the velocity is in the y direction at time t equals 0. There is no way around that. And how do you find now the velocity of the ball in the vertical direction, the velocity of the gun?

And that now follows from the angle theta here. I argue that if this is the velocity of the car, which you have just calculated, that the velocity of the bullet, the ball, of the gun, that this is v0 of the gun.

Only when this v0 of the gun has this value and when the car has this value of velocity of the car will you get this kind of a trajectory. Now once you recognize that, it is immediately obvious, it should be obvious to you that you cannot even tell the difference when you see the strobe pictures between your kicking a soccer ball at an angle theta and a car moving with constant velocity and shooting up the ball vertically as seen from the car. The two are mathematically completely identical. So once you recognize that you might as well use all the equations that we have derived for that case where we kick up the soccer ball. They're all at our finger tips. Equation 1 through 4 that we derived all holds. But those are not the only equations that hold. Equations that also hold are the equations that we found for t hits, for t h, for x hit, and for h itself. So what you have to do, you have to carefully calculate, measures as much as-- as accurate as you can the value for theta. You know this value. And so if you know theta and you know this value, you can calculate v0 in the y direction, and so you can now apply just as an example, I only take one example. If you now calculate how many seconds it takes for the ball to hit, that is 2 v0 sine theta divided by g.

Well, if you know this velocity and you know this velocity, then clearly, you know this velocity as well. This is v0 now. And so you know theta and you know v0, so you know v0, you know theta. You know t hits. This has been measured and this has been measured by you so out pops g. And that's what you had to calculate. You do not have to take t hit, you can also, for instance, take x hit. You can calculate what the distance is where the ball moves from the moment it's shot up to the moment it hits the car again. That equals v0 times the sine of 2 theta divided by g. You measure this distance. Do the best you can. You measure v0 and you have measured theta, so you know what the sine of 2 theta is. Out pops g.

If you prefer to measure h or if you prefer to measure t of h using your strobes, then again, in each one of these cases, will you be able to find g?

I want you to appreciate though, there's no way that you can bypass the velocity of the gun at time t equals 0. You must somehow measure that. So you're stuck to measuring this angle theta and deriving from that the speed of the gun. You cannot bypass that in any way.