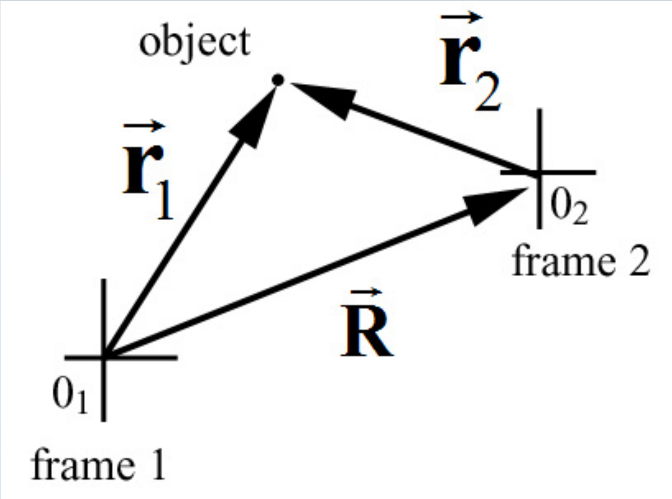
Kinematics in Two-Dimensions
8.01

Reference Systems

Use coordinate system as a ‘*reference frame*’ to describe the position, velocity, and acceleration of objects.

Relatively Inertial Reference Frames

Two reference frames.



Origins need not coincide.

One moving object has different position vectors in different frames

$$\vec{\mathbf{r}}_1 = \vec{\mathbf{R}} + \vec{\mathbf{r}}_2$$

Relative velocity between the two reference frames

$$\vec{\mathbf{V}} = d\vec{\mathbf{R}} \big/ dt$$

is constant since the relative acceleration is zero

$$\vec{\mathbf{A}} = d\vec{\mathbf{V}} \big/ dt = \vec{\mathbf{0}}$$

Law of Addition of Velocities

Suppose the object is moving; then, observers in different reference frames will measure different velocities

Velocity of the object in Frame 1: $\vec{\mathbf{v}}_1 = d\vec{\mathbf{r}}_1 / dt$

Velocity of the object in Frame 2: $\vec{\mathbf{v}}_2 = d\vec{\mathbf{r}}_2 / dt$

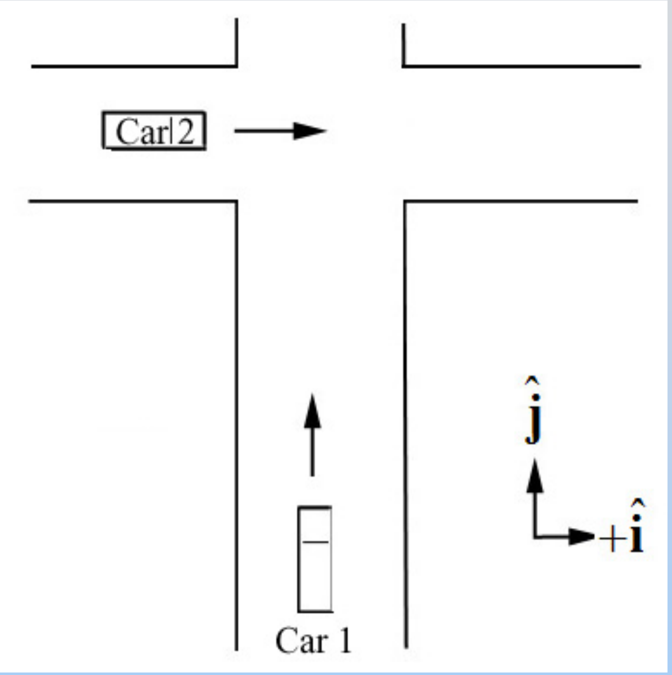
Velocity of an object in two different reference frames

$$\frac{d\vec{\mathbf{r}}_1}{dt} = \frac{\vec{\mathbf{R}}}{dt} + \frac{\vec{\mathbf{r}}_2}{dt}$$

$$\vec{\mathbf{v}}_1 = \vec{\mathbf{V}} + \vec{\mathbf{v}}_2$$

Checkpoint Problem

Suppose two cars, Car 1, and Car 2, are traveling along roads that are perpendicular to each other. Reference Frame 1 is at rest with respect to the ground. Reference Frame 2 is at rest with respect to Car 1. Choose unit vectors such that Car 1 is moving in the $+\hat{\mathbf{j}}$ direction, and Car 2 is moving in the $+\hat{\mathbf{i}}$ direction in reference Frame 1.



a) What is the vector description of the velocity of Car 2 in Reference Frame 2?

b) What is the magnitude of the velocity of Car 2 as seen in Reference Frame 2?

c) What angle does the velocity of Car 2 (as seen in Reference Frame 2) make with respect to the $+\hat{\mathbf{i}}$ direction?

Demo:

Relative Motion Gun A7

Video Link:

http://tsgphysics.mit.edu/front/index.php?page=demo.php?letnum=A%207&show=0

Summary Constant Acceleration

Components of Velocity:

$$v_x = v_{0,x} + a_x t \ , \qquad v_y = v_{0,y} + a_y t$$

Components of Position:

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2, \qquad y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$$

Eliminating t:

$$2a_x (x - x_0) = v_x^2 - v_{x,0}^2$$

Two-Dimensional Motion

Projectile Motion

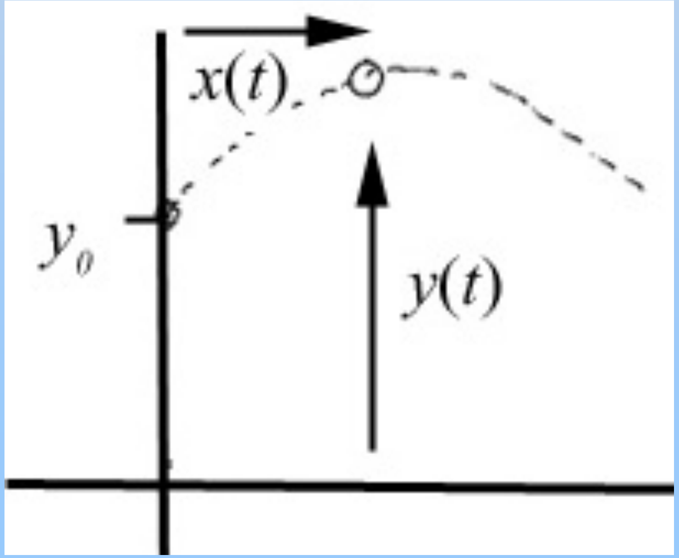
A projectile is fired from a height y_0 with an initial speed v_0 at an angle θ above the horizontal. Ignore air resistance

Gravitational Force Law

$$\vec{\mathbf{F}}_{\text{grav}} = -m_{\text{grav}} g\,\hat{\mathbf{j}} \quad \Rightarrow \quad F_{\text{grav},y} = -m_{\text{grav}} g$$

Newton's Second Law

$$\vec{\mathbf{F}}^{\text{total}} = m_{\text{in}} \vec{\mathbf{a}} \quad \Rightarrow \begin{cases} F_x^{\text{total}} = m_{\text{in}} a_x \\ F_y^{\text{total}} = m_{\text{in}} a_y \end{cases}$$



Equations of Motion

y-component: $-m_{\text{grav}} g = m_{\text{in}} a_y$

x-component: $0 = m_{\text{in}} a_x$

Principle of Equivalence: $m_{\text{grav}} = m_{\text{in}}$

Components of acceleration:

$a_x = 0, \;\; a_y = -g$ $\qquad g = 9.8 \text{ m/s}^2$

Checkpoint Problem: Softball

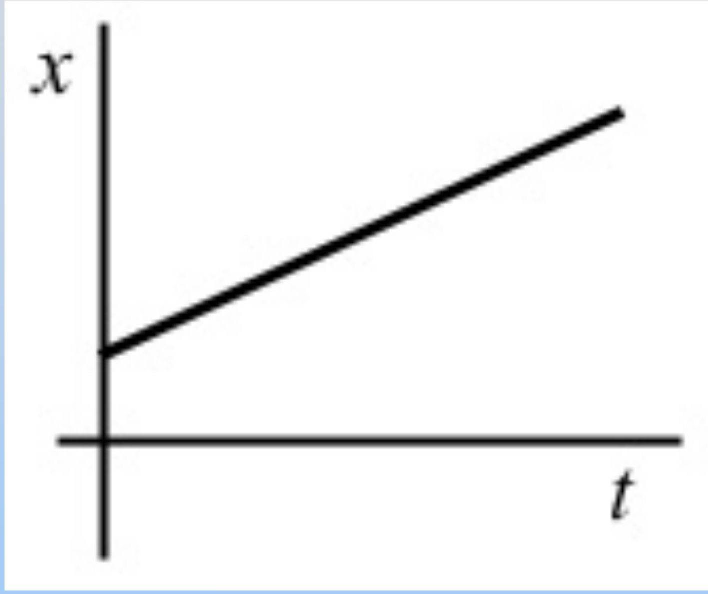
A softball is hit over a third baseman's head. The third baseman, as soon as the ball is hit, turns around and runs straight backwards a constant speed of 7 m/s for a time interval 2 s and catches the ball at the same height it left the bat. The third baseman was initially a distance 18 m from home plate. What was the initial speed and angle of the softball when it left the bat?

Kinematic Equations -- *x*-components :

Acceleration : $a_x = 0$

Velocity : $v_x(t) = v_{x,0}$

Position : $x(t) = x_0 + v_{x,0}t$

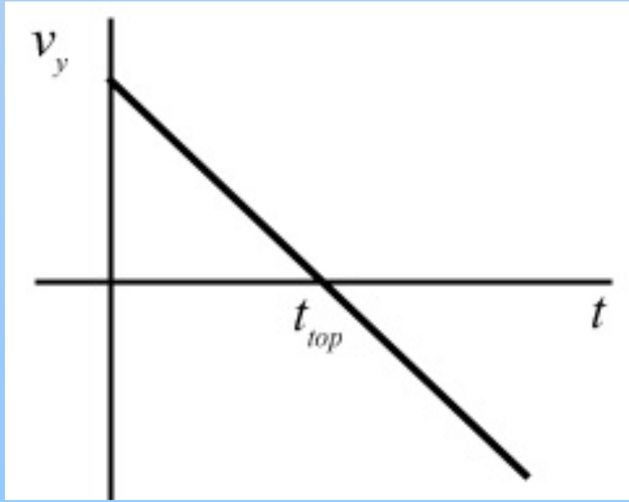


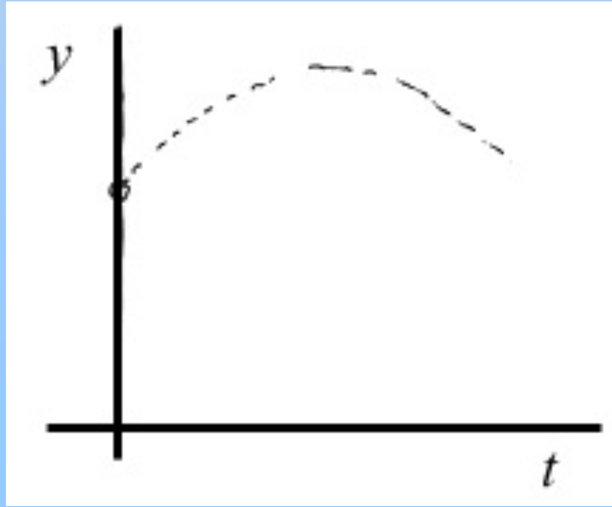
Kinematic Equations
y-components :

Acceleration: $a_y = -g$

Velocity: $v_y(t) = v_{y,0} - gt$

Position: $y(t) = y_0 + v_{y,0}t - \frac{1}{2}gt^2$





Vector Description of Motion

Position $$\vec{\mathbf{r}}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}}$$

Displacement $$\Delta\vec{\mathbf{r}}(t) = \Delta x(t)\,\hat{\mathbf{i}} + \Delta y(t)\,\hat{\mathbf{j}}$$

Velocity $$\vec{\mathbf{v}}(t) = \frac{dx(t)}{dt}\hat{\mathbf{i}} + \frac{dy(t)}{dt}\hat{\mathbf{j}} \equiv v_x(t)\hat{\mathbf{i}} + v_y(t)\hat{\mathbf{j}}$$

Acceleration $$\vec{\mathbf{a}}(t) = \frac{dv_x(t)}{dt}\hat{\mathbf{i}} + \frac{dv_y(t)}{dt}\hat{\mathbf{j}} \equiv a_x(t)\hat{\mathbf{i}} + a_y(t)\hat{\mathbf{j}}$$

Checkpoint Problem: Baseball

A batter hits a baseball into the air with an initial speed, v_0 = 50 m/s , and makes an angle θ_0 = 30^o with respect to the horizontal. How far does the ball travel if it is caught at exactly the same height that it is hit from? When the ball is in flight, ignore all forces acting on the ball except for gravitation.

Initial Conditions

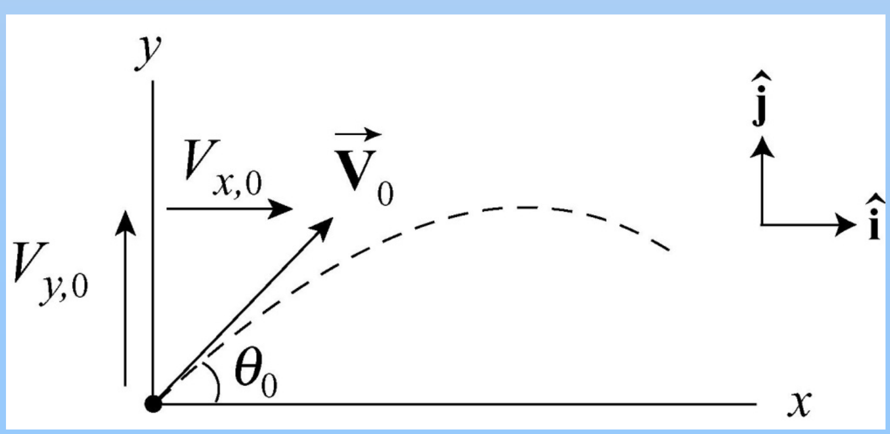
- Initial position: $\vec{\mathbf{r}}_0 = x_0\hat{\mathbf{i}} + y_0\hat{\mathbf{j}}$

- Initial velocity: $\vec{\mathbf{v}}_0(t) = v_{x,0}\hat{\mathbf{i}} + v_{y,0}\hat{\mathbf{j}}$

- Velocity components: $v_{x,0} = v_0 \cos\theta_0, \quad v_{y,0} = v_0 \sin\theta_0$

- Initial speed: $v_0 = |\vec{\mathbf{v}}_0| = \left(v_{x,0}^2 + v_{y,0}^2\right)^{1/2}$

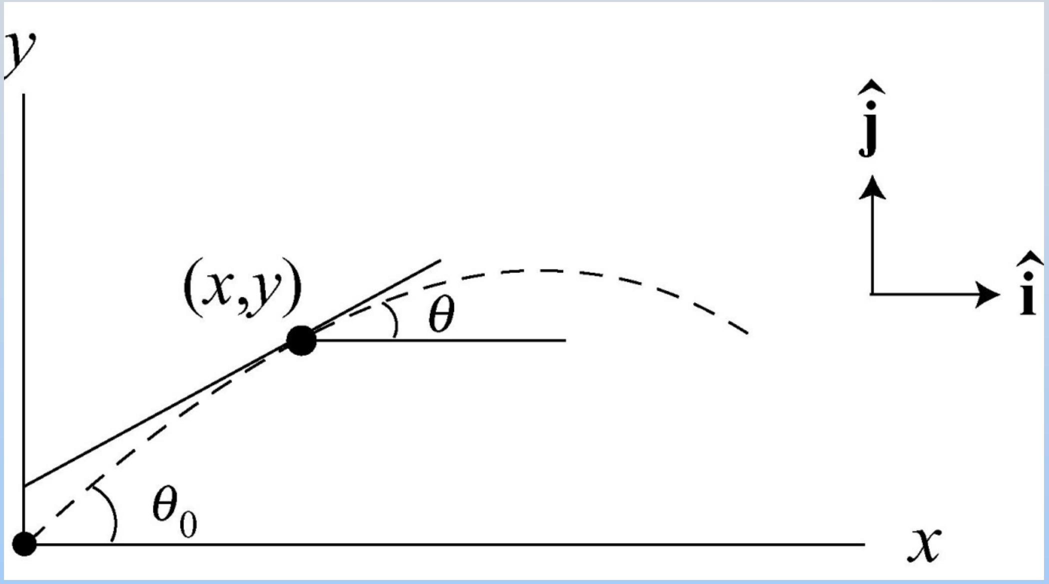
- Direction: $\theta_0 = \tan^{-1}\left(\dfrac{v_{y,0}}{v_{x,0}}\right)$



Orbit equation

The slope of the curve $y(t)$ vs. $x(t)$ at any point determines the direction of the velocity

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)$$



$$(x_0, y_0) = (0,0)$$

$$x(t) = v_{x,0}t \quad \Rightarrow \quad t = \frac{x(t)}{v_{x,0}}$$

$$y(t) = v_{y,0}t - \frac{1}{2}gt^2 = \frac{v_{y,0}}{v_{x,0}}x(t) - \frac{1}{2}\frac{g}{v_{x,0}^2}x(t)^2$$

Checkpoint Problem: Stuffed Animal and the Gun

A stuffed animal is suspended at a height h above the ground. A physics demo instructor has set up a projectile gun a horizontal distance d away from the stuffed animal. The projectile is initially a height s above the ground. The demo instructor fires the projectile with an initial velocity of magnitude v_0 just as the stuffed animal is released. Find the angle the projectile gun must be aimed in order for the projectile to strike the stuffed animal. Ignore air resistance.

Demo:

Stuffed Animal and Gun A6

Video Link

http://tsgphysics.mit.edu/front/index.php?page=demo.php?letnum=A%207&show=0

Checkpoint Problem: Throwing a Stone Down a Hill

A person is standing on top of a hill that slopes downwards uniformly at an angle ϕ with respect to the horizontal. The person throws a stone at an initial angle θ_0 from the horizontal with an initial speed v_0 . You may neglect air resistance. How far below the top of the hill does the stone strike the ground?