Two Dimensional Kinematics
Challenge Problems

Problem 1:

Suppose a MIT student wants to row across the Charles River. Suppose the water is moving downstream at a constant rate of 1.0 m/s. A second boat is floating downstream with the current. From the second boat's viewpoint, the student is rowing perpendicular to the current at 0.5 m/s. Suppose the river is 800 m wide.

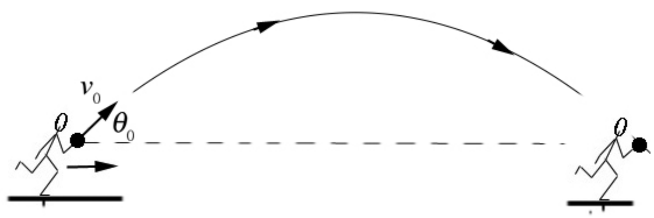
a) What is the direction and magnitude of the velocity of the student as seen from an observer at rest along the bank of the river?

b) How far down river does the student land on the opposite bank?

c) How long does the student take to reach the other side?

Problem 2:

A person initially at rest throws a ball upward at an angle θ_0 with an initial speed v_0. He tries to catch up to the ball by accelerating with a constant acceleration a for a time interval Δt_1 and then continues to run at a constant speed for a time interval Δt_2. He catches the ball at exactly the same height he threw the ball. Let g be the gravitational constant. What was the person's acceleration a?

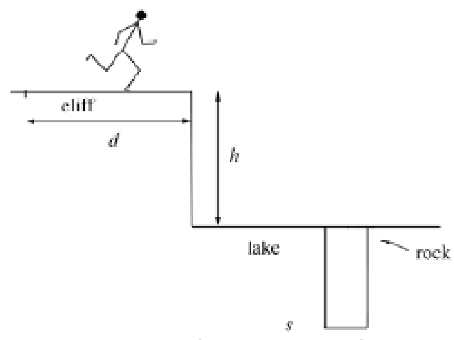


Problem 3:

A person, standing on a vertical cliff a height h above a lake, wants to jump into the lake but notices a rock just at the surface level with its furthest edge a distance s from the shore. The person realizes that with a running start it will be possible to just clear the rock, so the person steps back from the edge a distance d and starting from rest, runs at an acceleration that varies in time according to

$$a_x = b_1 t$$

and then leaves the cliff horizontally. The person just clears the rock. Find s in terms of the given quantities d, b_1, h, and the gravitational constant g. You may neglect all air resistance.



Problem 4:

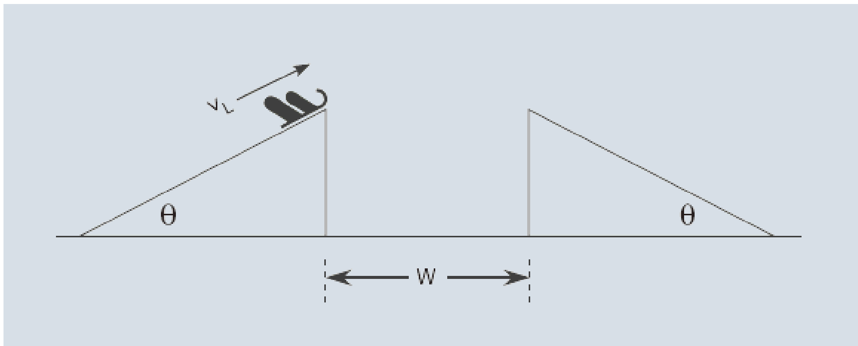
A bicyclist has an acceleration in the x-direction given by

$$a_x = b_1 t^2, \quad 0 \le t \le t_1$$

where $b_1 > 0$. At $t = 0$ the bicyclist is located at $x_0 = 0$ and starts from rest. At that same instant ($t = 0$) a ball, located along the path of motion of the bicycle at $x_{ball}(t = 0) = d$, is thrown vertically upwards with a non-zero y-component of velocity given by v_{y0}. At some later time $t = t_1$ the bicyclist catches the ball on the way down from the same height that is was thrown. You may ignore any effects of air resistance. Find an expression for b_1 in terms of d, v_{y0}, and g but do not use t_1 in your answer. What are the dimensions of b_1?

Problem 5:

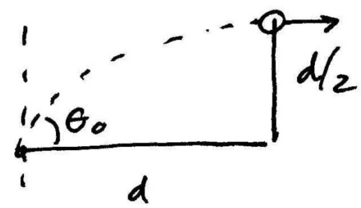
Calvin and Hobbes are riding on a sled. They are trying to jump the gap between two symmetrical ramps of snow separated by a distance W as shown above. Each ramp makes an angle θ with the horizontal. They launch off the first ramp with a speed v_L. Calvin, Hobbs and the sled have a total mass m.



a) What value of the initial launch speed v_L will result in the sled landing exactly at the peak of the second ramp? Express your answer in terms of some (or all) of the parameters m, θ, W, and the acceleration of gravity g. Include in your answer a brief description of the strategy that you used and any diagrams or graphs that you have chosen for solving this problem. Make sure you clearly state which concepts you plan to use to calculate any relevant physical quantities.

Problem 6:

A person is a playing a game that requires throwing an object onto a ledge. The ledge is a distance d and a height $d/2$ above the release point. You may neglect air resistance. You may use g for the magnitude of the gravitational acceleration.

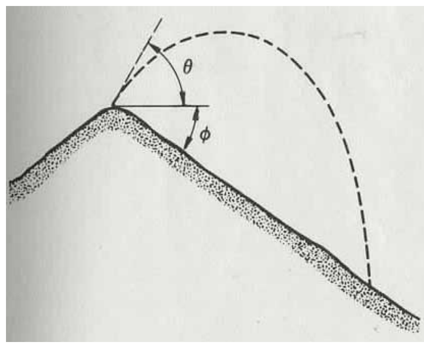


(a) At what angle must the person throw the object and with what magnitude of the velocity if the object is to be exactly at the top of its flight when it reaches the ledge? Briefly describe how you will model this problem and your strategy for finding the answer. Express your answer in terms of the given quantities d and g, as needed.

(b) What is the horizontal component of the acceleration? Again briefly describe your model and strategy for solving the problem. Express your answer in terms of the given quantities s, d, and g.

Problem 7: Two-Dimensional Kinematics: *Projectile motion*

A person is standing on top of a hill that slopes downwards uniformly at an angle ϕ with respect to the horizontal. The person throws a stone at an initial angle θ_0 from the horizontal with an initial speed of v_0. You may neglect air resistance. What is the horizontal range of the stone when the stone strikes the ground?



Problem 8: Instructions to Shoot Down a Rising Weather Balloon

You must give the crew of a howitzer instructions for shooting down an enemy weather balloon (diameter $d = 5\ \text{m}$) launched a known distance ($D = 5\ \text{km}$ to $10\ \text{km}$) from their gun. They can control the velocity and angle of their gun within the limit that it has a 25 km range (but can be fired at a controllably lower velocity) and maximum elevation of 45 degrees, and can fire the gun any time. It takes 30 seconds to change the speed of the projectile (aiming takes zero time; it can track a target's motion), and 50 seconds for loading or reloading. After the balloon has risen 5 meters, it rises at a constant speed v_B, which is typically in the range 100 to 300 meters per minute. Try to come up with as simple instructions as possible. You can assume the gun crew has optical surveying equipment. The weather team does not launch balloons if the wind is over 0.3 meters per second.

Write as simple instructions as possible, and show calculations that indicate the projectile will hit the target and that you have considered and solved the likely complications.

Problem 9: Hitting the Bucket

A person is standing on a ladder holding a pail. The person releases the pail from rest at a height above the ground. A second person standing a horizontal distance s_2 from the pail aims and throws a ball the instant the pail is released in order to hit the pail. The person throws the ball at a height h_2 above the ground, with an initial speed v_0, and at an angle θ_0 with respect to the horizontal. You may ignore air resistance.

Questions:

a) Find an expression for the angle θ_0 that the person aims the ball in order to hit the pail as a function of the other variables given in the problem

b) If the person aims correctly, find an expression for the range of speeds that the ball must be thrown at in order to hit the pail?

c) Find an expression for the angle θ_0 that the person throws the ball as a function of h_1, h_2, and s_2.

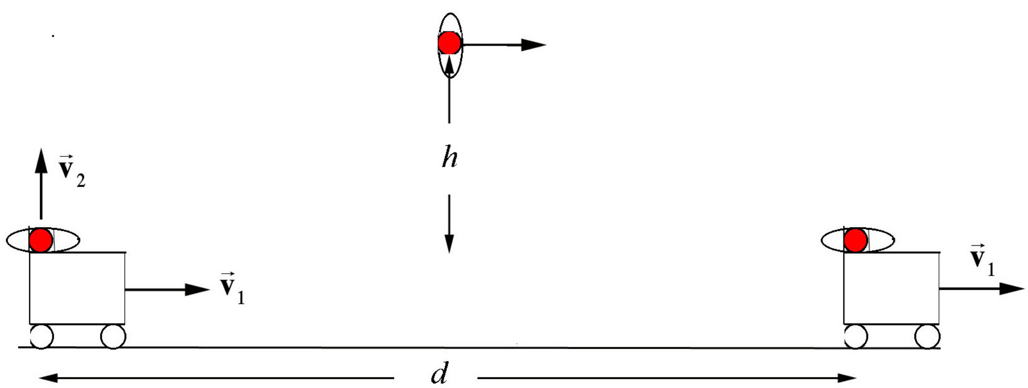
d) Find an expression for the time of collision as a function of the initial speed of the ball v_0, and the quantities h_1, h_2, and s_2.

e) Find an expression for the height above the ground where the collision occurred as a function of the initial speed of the ball v_0, and the quantities h_1, h_2, and.

f) Find an expression for the range of speeds (as a function of h_1, h_2, and s_2) that the ball can be thrown in order that the ball will collide with the pail?

Problem 10: Moving cart and hoop Kinematics

A cart moving with horizontal velocity $\vec{\mathbf{v}}_1$ with magnitude v_1 ejects a ball with a velocity $\vec{\mathbf{v}}_2$ (magnitude v_2) in the vertical direction relative to the moving cart. The ball must pass horizontally through a small stationary hoop appropriately oriented and located at a height h above the launch point. Later the ball passes through a second hoop that is fixed horizontally on the cart and centered at the launch point. Let g be the gravitational acceleration. Express your answers in terms of the given quantities v_1, v_2, and g. You may ignore air resistance, and any rolling friction on the cart.



a) Why will the ball pass through the hoop mounted on the cart as it comes back down? Briefly explain your answer.

b) Find an expression for the time interval it takes the ball to move from its launch point to the center of the top hoop. You may neglect the thickness of the hoop. Show all your work.

c) Find an expression for the vertical distance h between the center of top hoop and the launch point. Show all your work.

d) Find an expression for the horizontal distance d the ball has traveled relative to the ground when it returns to its launch point on the cart. Show all your work.