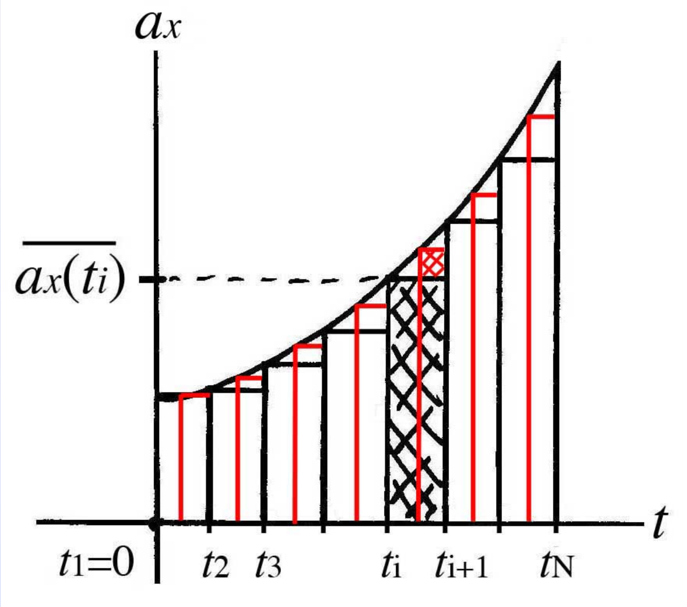
One Dimensional Kinematics
Non-Constant Acceleration

Velocity as the Integral of Acceleration

the area under the graph of the *x*-component of the acceleration vs. time is the change in velocity



$$\int_{t'=0}^{t'=t} a_x(t')dt' \equiv \lim_{\Delta t_i \to 0} \sum_{i=1}^{i=N} a_x(t_i)\Delta t_i = Area(a_x, t)$$

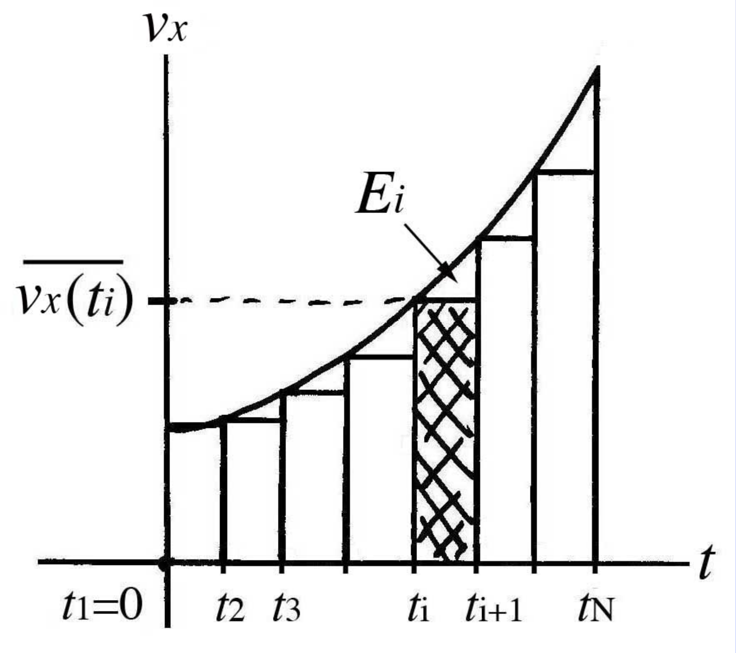
$$\int_{t'=0}^{t'=t} a_x(t')dt' = \int_{t'=0}^{t'=t} \frac{dv_x}{dt} dt' = \int_{v'_x=v_x(t=0)}^{v'_x=v_x(t)} dv'_x = v_x(t) - v_{x,0}$$

Position as the Integral of Velocity

area under the graph of x-component of the velocity vs. time is the displacement

$$v_x(t) \equiv \frac{dx}{dt}$$

$$\int_{t'=0}^{t'=t} v_x(t')dt' = x(t) - x_0$$



Summary: Time Dependent Acceleration

- Acceleration is a non-constant function of time

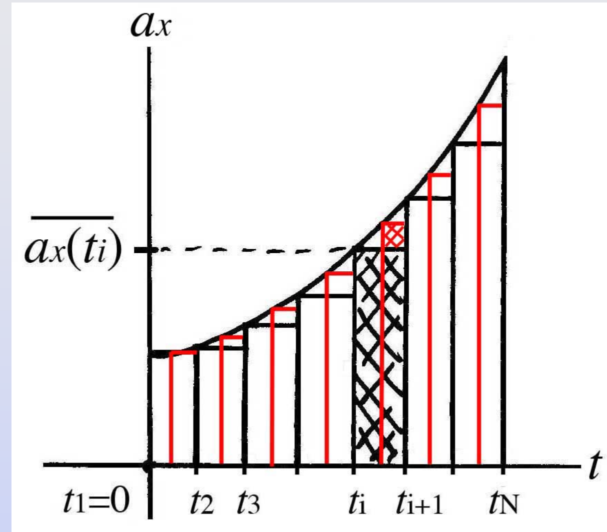
$$a_x(t)$$

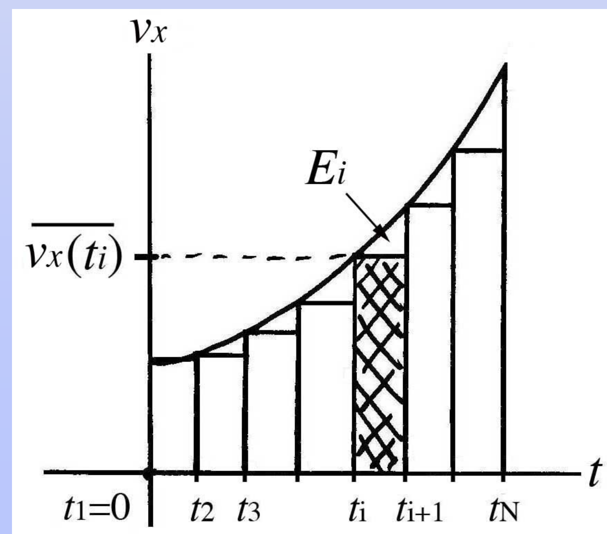
- Change in velocity

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t')dt'$$

- Change in position

$$x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t')dt'$$

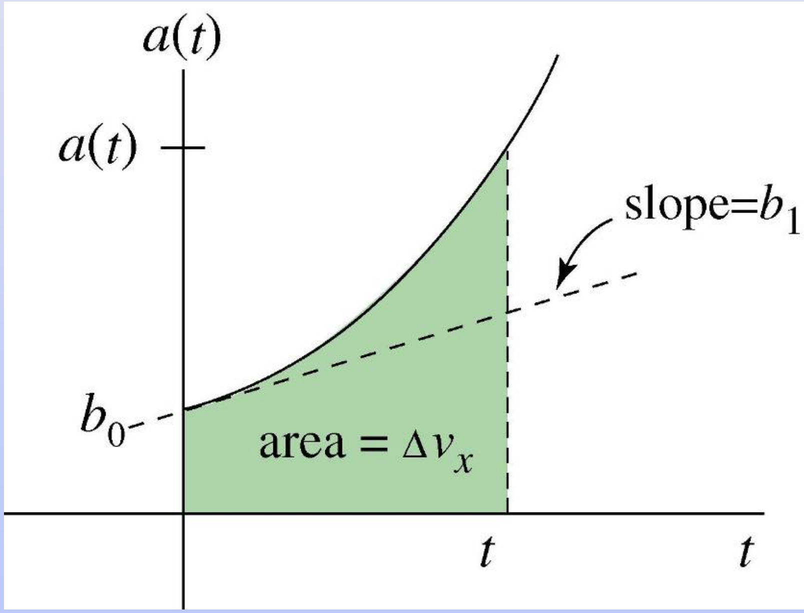




Worked Example: Non-constant acceleration

- Consider an object released at time $t = 0$ with an initial *x*-component of velocity $v_{x,0}$, located at position x_0 , and accelerating according to

$$a_x(t) = b_0 + b_1 t + b_2 t^2$$



- Find the velocity and position as a function of time.

Worked Example: Non-constant acceleration

Velocity:

$$v_x(t) = v_{x,0} + \int_{t'=0}^{t'=t} a_x(t')dt'$$

$$= v_{x,0} + \int_{t'=0}^{t'=t} (b_0 + b_1 t' + b_2 t^2)dt' = v_{x,0} + b_0\, t'\Big|_{t'=0}^{t'=t} + \frac{1}{2} b_1\, t'^2\Big|_{t'=0}^{t'=t} + \frac{1}{3} b_2\, t'^3\Big|_{t'=0}^{t'=t}$$

$$= v_{x,0} + b_0 t + \frac{1}{2} b_1 t^2 + \frac{1}{3} b_2 t^3$$

Position:

$$x(t) = x_0 + \int_{t'=0}^{t'=t} v_x(t')dt' =$$

$$= x_0 + \int_{t'=0}^{t'=t} (v_{x,0} + b_0 t' + \frac{1}{2} b_1 t'^2 + \frac{1}{3} b_2 t'^3)dt' = v_{x,0} t + \frac{1}{2} b_0 t^2 + \frac{1}{6} b_1 t^3 + \frac{1}{12} b_2 t'^4$$

Checkpoint Problem: Non-Constant Acceleration:

Consider an object released at time $t = 0$ with an initial *x*-component of velocity $v_{x,0}$, located at position x_0 and accelerating according to

$$a_x(t) = b_0 - b_1 t$$

Find the velocity and position as a function of time.

Checkpoint Problem: Sports Car

At $t = 0$, a sports car starting at rest at $x = 0$ accelerates with an x-component of acceleration given by

$$a_x(t) = \alpha t - \beta t^3, \text{ for } 0 < t < (\alpha/\beta)^{1/2}.$$

and zero afterwards with $\alpha, \beta > 0$.

(1) Find expressions for the velocity and position vectors of the sports as functions of time for $0 < t < (\alpha/\beta)^{1/2}$

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(2) Sketch graphs of the x-component of the position, velocity and acceleration of the sports car as a function of time for $0 < t < (\alpha/\beta)^{1/2}$.

8.01SC Physics I: Classical Mechanics