Kinematics and One Dimensional Motion

Kinematics Vocabulary

- *Kinema* means movement
- Mathematical description of motion
  - Position
  - Time Interval
  - Displacement
  - Velocity; absolute value: speed
  - Acceleration
  - Averages of the later two quantities.

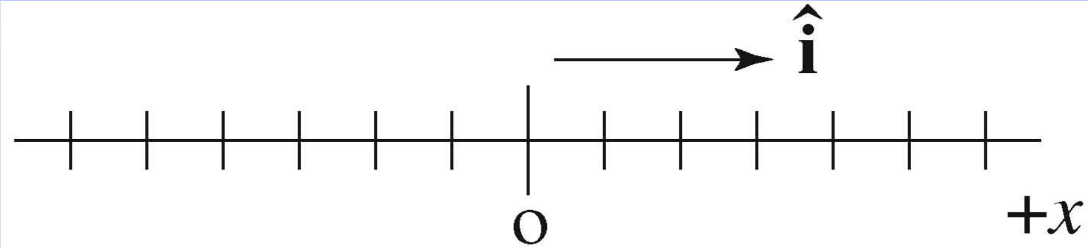
Coordinate System in One Dimension

Used to describe the position of a point in space

A coordinate system consists of:

1. An origin at a particular point in space
2. A set of coordinate axes with scales and labels
3. Choice of positive direction for each axis: unit vectors
4. Choice of type: Cartesian or Polar or Spherical

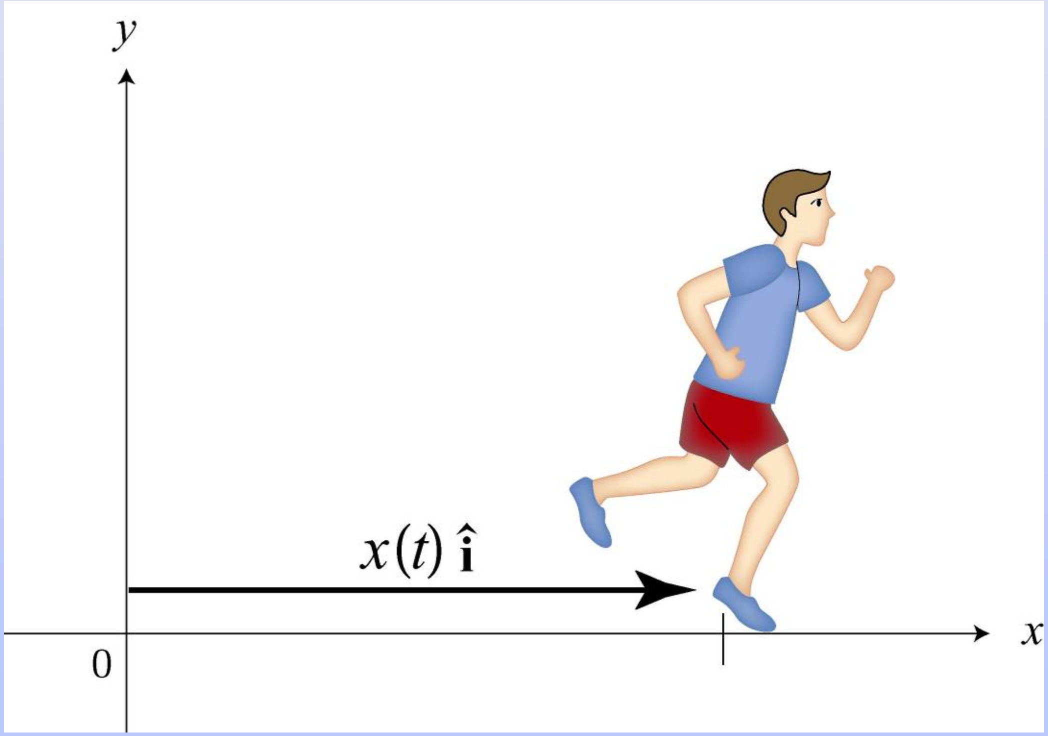
Example: Cartesian One-Dimensional Coordinate System



Position

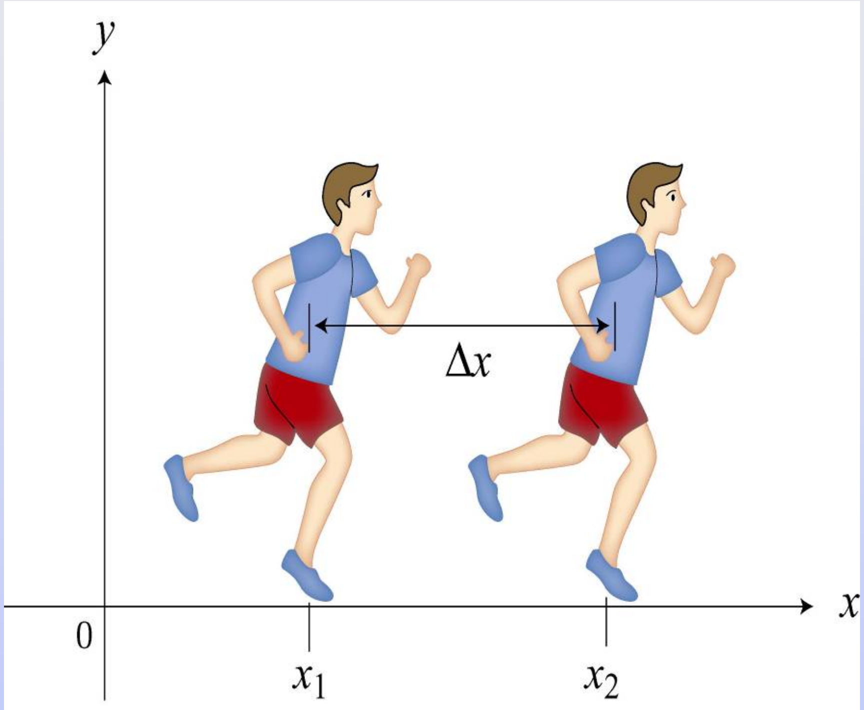
- A vector that points from origin to body.
- Position is a function of time
- In one dimension:

$$\vec{\mathbf{x}}(t) = x(t)\hat{\mathbf{i}}$$



Displacement Vector

Change in position vector of the object during the time interval $\Delta t = t_2 - t_1$



$$\Delta\vec{\mathbf{r}} \equiv \left(x(t_2) - x(t_1) \right)\hat{\mathbf{i}} \equiv \Delta x(t)\hat{\mathbf{i}}$$

Average Velocity

The average velocity, $\overline{\vec{\mathbf{v}}}(t)$, is the displacement $\Delta\vec{\mathbf{r}}$ divided by the time interval Δt

$$\overline{\vec{\mathbf{v}}}(t) \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\mathbf{i}} = \overline{v_x}(t)\hat{\mathbf{i}}$$

The x-component of the average velocity is given by

$$\overline{v_x}(t) = \frac{\Delta x}{\Delta t}$$

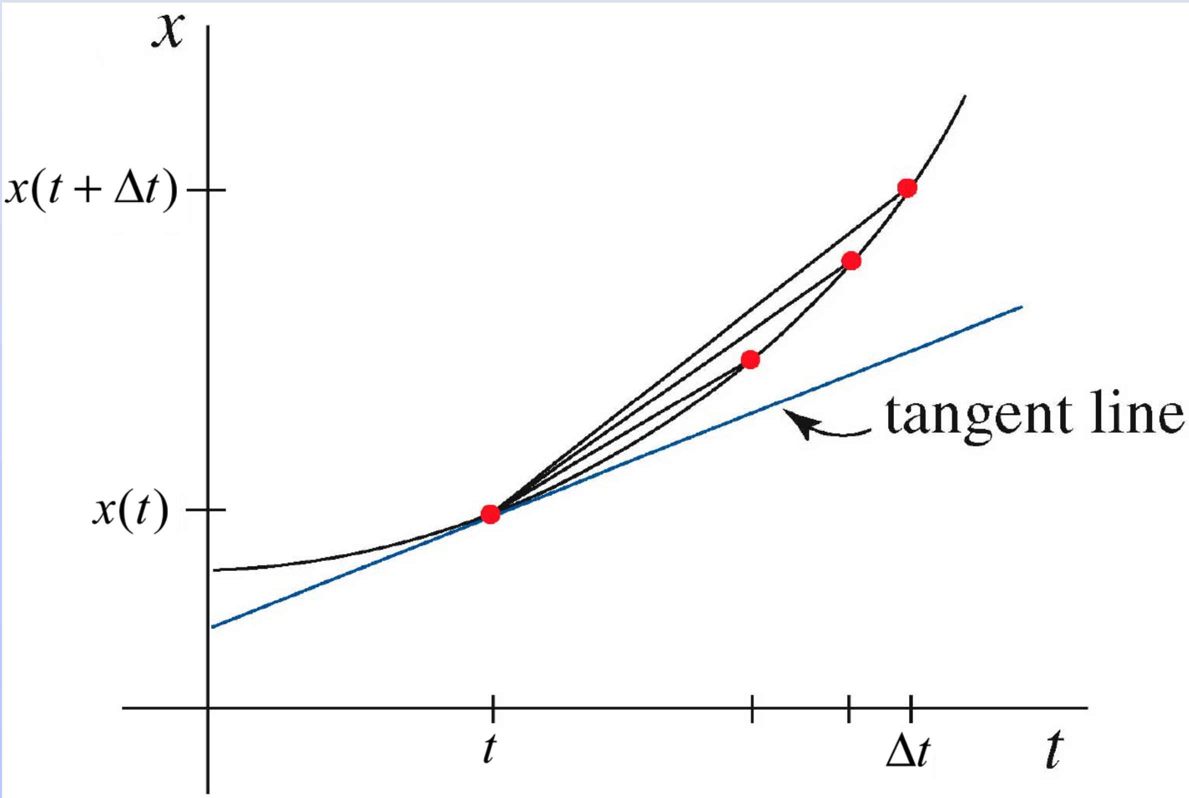
Instantaneous Velocity

For each time interval Δt, we calculate the *x*-component of the average velocity. As $\Delta t \to 0$, we generate a sequence of the *x*-component of the average velocities. The limiting value of this sequence is defined to be the *x*-component of the instantaneous velocity at the time t.

$$v_x(t) \equiv \lim_{\Delta t \to 0} \overline{v}_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \frac{x\left(t + \Delta t\right) - x\left(t\right)}{\Delta t} \equiv \frac{dx}{dt}$$

Instantaneous Velocity

x-component of the velocity is equal to the slope of the tangent line of the graph of x-component of position vs. time at time t



Average Acceleration

Change in instantaneous velocity divided by the time interval $\Delta t = t_2 - t_1$

$$\vec{\bar{\mathbf{a}}} \equiv \frac{\Delta \vec{\mathbf{v}}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{\mathbf{i}} = \frac{(v_{x,2} - v_{x,1})}{\Delta t}\hat{\mathbf{i}} = \frac{\Delta v_x}{\Delta t}\hat{\mathbf{i}} = \overline{a_x}\,\hat{\mathbf{i}}$$

The *x*-component of the average acceleration

$$\overline{a_x} = \frac{\Delta v_x}{\Delta t}$$

Instantaneous Acceleration

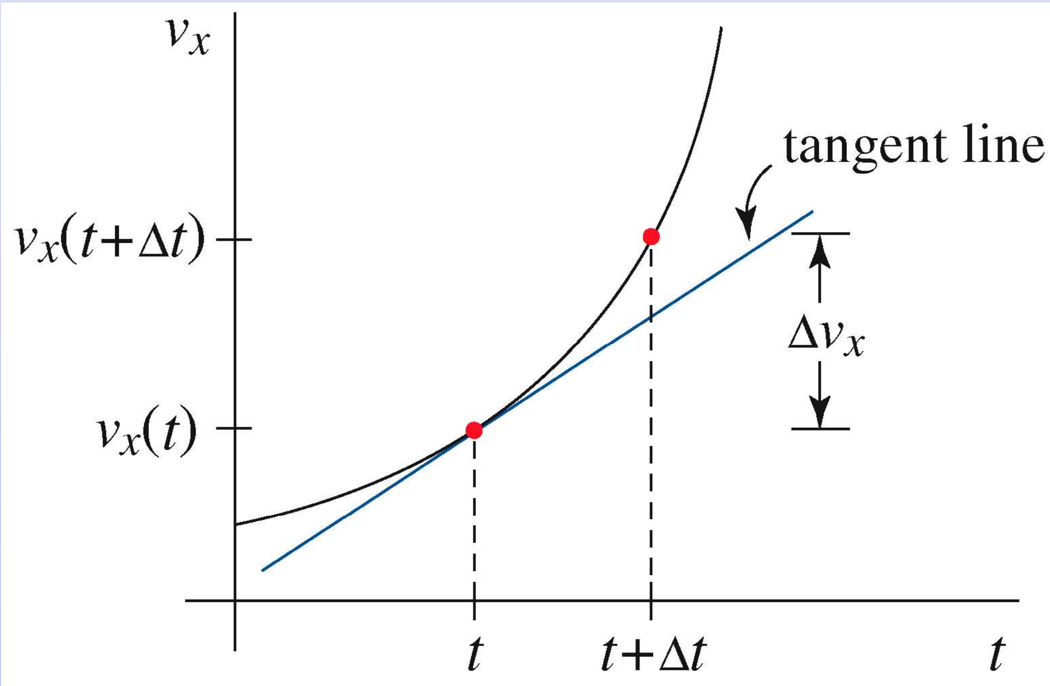
For each time interval Δt, we calculate the *x*-component of the average acceleration. As $\Delta t \to 0$, we generate a sequence of *x*-component of average accelerations. The limiting value of this sequence is defined to be the *x*-component of the instantaneous acceleration at the time t.

$$\vec{\mathbf{a}}(t) = a_x(t)\hat{\mathbf{i}} \equiv \lim_{\Delta t \to 0} \overline{a_x}\hat{\mathbf{i}} = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}\hat{\mathbf{i}} = \lim_{\Delta t \to 0} \frac{v_x(t+\Delta t) - v_x(t)}{\Delta t}\hat{\mathbf{i}} \equiv \frac{dv_x}{dt}\hat{\mathbf{i}}$$

$$a_x(t) = \frac{dv_x}{dt}$$

Instantaneous Acceleration

The *x*-component of acceleration is equal to the slope of the tangent line of the graph of the *x*-component of the velocity vs. time at time *t*



Checkpoint Problem: Model Rocket

A person launches a home-built model rocket straight up into the air at $y = 0$ from rest at time $t = 0$. (The positive y-direction is upwards). The fuel burns out at $t = t_0$. The position of the rocket is given by

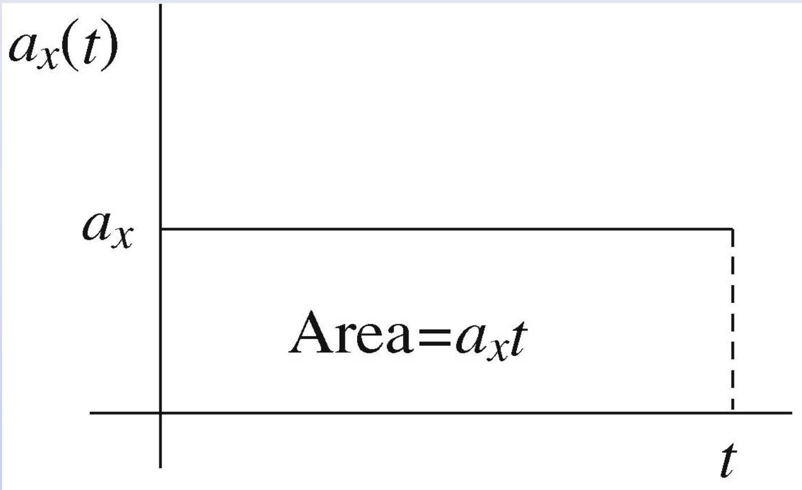
$$y = \begin{cases} \frac{1}{2}(a_0 - g)t^2 - \frac{a_0}{30} t^6 / t_0{}^4; \qquad 0 < t < t_0 \end{cases}$$

with a_0 and g are positive. Find the y-components of the velocity and acceleration of the rocket as a function of time. Graph a_y vs t for $0 < t < t_0$.

Constant Acceleration: area under the acceleration vs. time graph

$$a_x = \overline{a_x} = \frac{\Delta v_x}{\Delta t} = \frac{v_x(t) - v_{x,0}}{t}$$

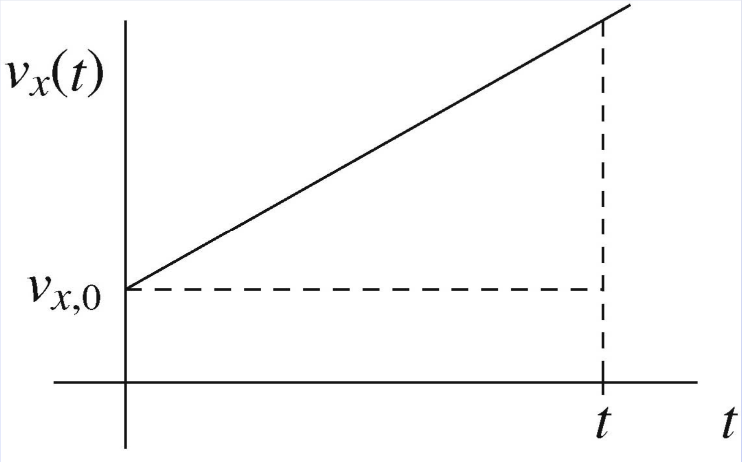
$$v_x(t) = v_{x,0} + a_x t$$



Constant Acceleration: area under the velocity vs. time graph

$$\text{Area}(v_x, t) = v_{x,0}t + \frac{1}{2}(v_x(t) - v_{x,0})t$$

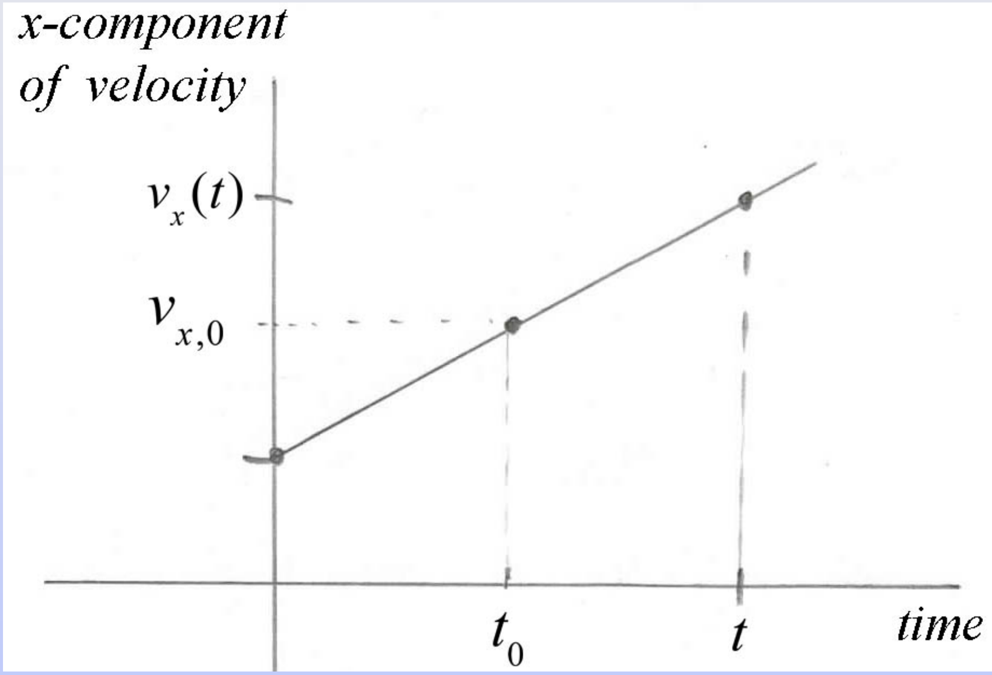
$$v_x(t) = v_{x,0} + a_x t$$



$$\text{Area}(v_x, t) = v_{x,0}t + \frac{1}{2}(v_{x,0} + a_x t - v_{x,0})t = v_{x,0}t + \frac{1}{2}a_x t^2$$

Constant Acceleration: average velocity

When the acceleration is constant, the velocity is a linear function of time. Therefore the average velocity is



$$\overline{v_x} = \frac{1}{2}\left(v_x(t) + v_{x,0}\right)$$

$$\overline{v_x} = \frac{1}{2}\left(v_x(t) + v_{x,0}\right) = \frac{1}{2}\left(\left(v_{x,0} + a_x t\right) + v_{x,0}\right) = v_{x,0} + \frac{1}{2} a_x t$$

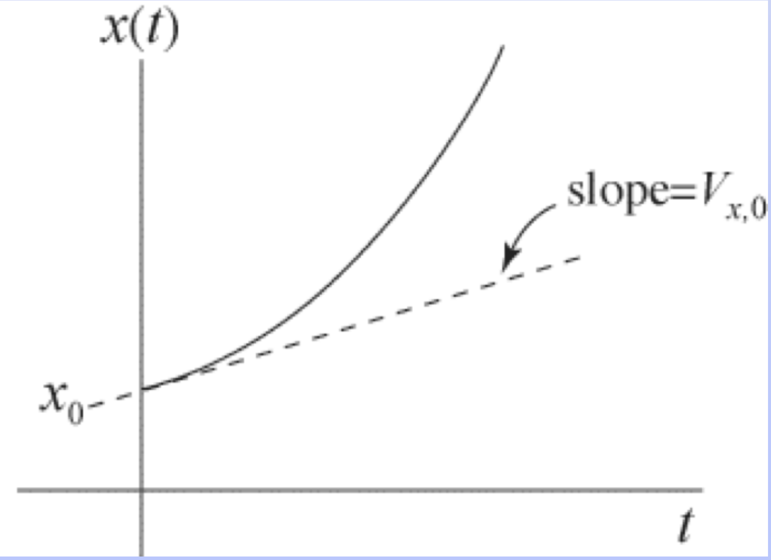
Constant Acceleration: area under the velocity vs. time graph

Displacement is equal to the area under the graph of the *x*-component of the velocity vs. time

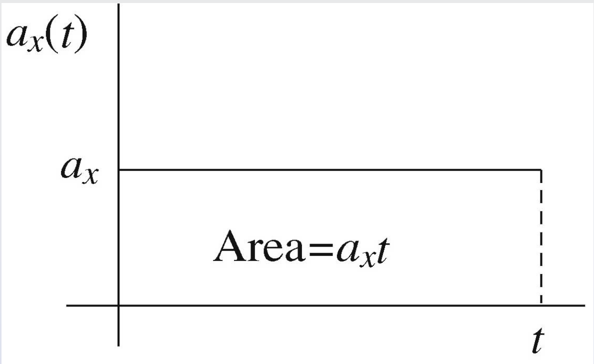
$$\Delta x \equiv x(t) - x_0 = \overline{v_x} t = v_{x,0} t + \frac{1}{2} a_x t^2$$

$$\Delta x = Area(v_x, t)$$

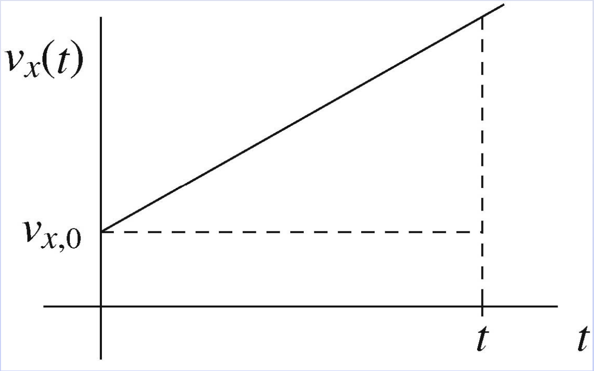
$$x(t) = x_0 + v_{x,0} t + \frac{1}{2} a_x t^2$$



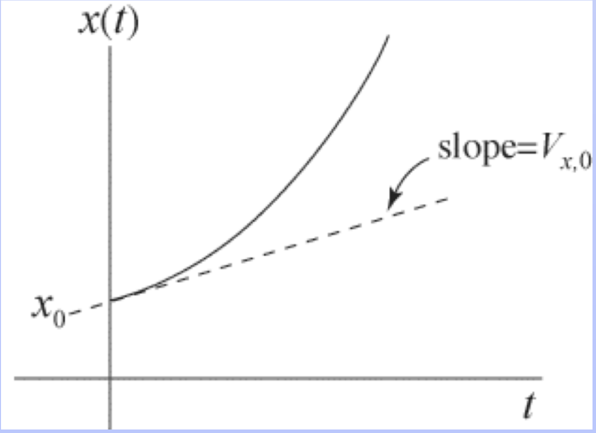
Summary: Constant Acceleration



- Acceleration $a_x = constant$



- Velocity $v_x(t) = v_{x,0} + a_x t$



- Position $x(t) = x_0 + v_{x,0}t + \frac{1}{2}a_x t^2$

Problem Solving Strategies: One-Dimensional Kinematics

Strategy: One Dimensional Kinematics

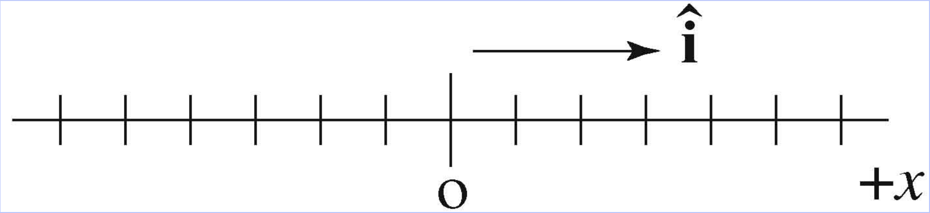
1. Define an appropriate coordinate system

2. Integrate or differentiate

A coordinate system consists of:

   1. An origin at a particular point in space
   2. A set of coordinate axes with scales and labels
   3. Choice of positive direction for each axis with a set of unit vectors
   4. Choice of type: Cartesian or Polar or Spherical

Example: Cartesian One-Dimensional Coordinate System



Checkpoint Problem: Runner

A runner accelerates from rest with a constant *x*-component of acceleration $2.0\ m\ s^{-2}$ for 2.0 s and then travels at a constant velocity for an additional 6.0 s. How far did the runner travel?

Checkpoint Problem: One Dimensional Kinematics

Bus and car

At the instant a traffic light turns green, a car starts from rest with a given constant acceleration, $5.0 \times 10^{-1}\ \mathrm{m \cdot s^{-2}}$. Just as the light turns green, a bus, traveling with a given constant speed, $1.6 \times 10^{1}\ \mathrm{m \cdot s^{-1}}$, passes the car. The car speeds up and passes the bus some time later. How far down the road has the car traveled, when the car passes the bus?

Checkpoint Problem: Motorcycle and Car

A car is driving at a constant but unknown velocity on a straightaway. A motorcycle is a distance *d* behind the car. Initially, the vehicles are both traveling at the same speed v_0. The motorcycle starts to pass the car by speeding up with a constant acceleration of magnitude a_m. When the motorcyclist is side by side with the car, the motorcycle stops accelerating and is traveling at twice the velocity of the car. The problem is to determine how far the motorcycle travels while accelerating and is side by side with the car. Express all your answers to the questions below in terms of the given quantities *d* and the magnitude of the acceleration a_m. (Hint: You may need to determine some other quantities before you can arrive at your final answer.)

Gravitational Free Fall

- Choose coordinate system with *x*-axis vertical, origin at ground, and positive unit vector pointing upward.
- Acceleration: $$a_x = -g = -(9.8\ \mathrm{m} \cdot \mathrm{s}^{-2})$$
- Velocity $$v_x(t) = v_{x,0} - gt$$
- Position: $$x(t) = x_0 + v_{x,0}t - \frac{1}{2}gt^2$$