Now we go to 9.4.6.

An airplane is landing with velocity v0. Here is the wheel of the airplane-- it has a mass m, it has radius R, and omega equals 0. The wheels are not rotating. Every point of the wheel has this forward velocity v 0 at the center of mass and the bottom. As it touches down, there will be a frictional force because it's slipping with the runway, and this frictional force will be a maximum at touchdown-- this is before, just at the moment of touchdown. There is a moment of inertia about this axis-- about this rotation axis-- which is I.

Let's look at the situation after this wheel has come to what we call a pure roll. What does it mean that it has come to a pure roll? Here is that wheel, and here is the runway. If this wheel has made one full rotation-- I'll make a little mark here, and this mark appears here again-- if the distance between the center of mass equals 2 pi R, and this is R, then we call this per definition complete pure rotation, or pure roll. There is no slip.

You can easily see that, because the circumference here equals 2 pi R. The time to go around, T-- that is the period to go around-- is 2 pi divided by omega, and so the velocity of the circumference is obviously 2 pi R divided by the time that it goes around. The velocity of the center of mass must be equal to omega times R.

The velocity of the center of mass-- this is the velocity of center of mass-- is 2 pi R. That's the distance that it moved, divided by the time for this object to go around. 2 pi divided by T is omega. You see here that the pure roll condition, and the sufficient condition, is that this v of the center of mass must be omega R. That is the pure roll condition, and then we have no longer any slip. We have an angular velocity omega now, and this radius is R.

You can do the same problem by taking a ball-- a billiard ball-- and you can shove the billiard ball on the table so that there is no rolling action, and then after awhile you will see that the billiard ball is going to go into a pure roll situation. It may not be so easy to demonstrate to you, but I have a ball here. If I were to move the ball to slide it along so that I have exactly this situation, then when I do that and let it go for a while, it goes into pure roll. It's already in pure roll, and goes very quickly in this case. You see I'm shoving it, and it is the friction that torques it up to go into pure roll.

Let's massage this further, which is not so easy. In fact, I was not able to do this problem in a very simple manner. I'm trying to find my drawing-- where is my drawing? I will have to make a new drawing, because I can't find my drawing

When it touches down, there is this frictional force, and this is the radius R. First of all, we have to obey Newton-- F, which is this force, so I will write it without this force, equals m times a times the center of mass. That's nonnegotiable-- that always must hold. v of the center of mass must be v 0 minus at, which is the same a. Notice that this minus sign already takes into account that the wheel is going to be decellerate-- in other words, a as a result will be positive, because the minus sign is already built in.

Then I have the v center of mass, which at any moment in time that I am in pure roll, must be omega t times R-- that's my pure roll condition. Then I have this tau that is relative to point C that equals R times F-- torque is in this direction, and that's why it will try to make omega increase in this direction. That is R times F, and that is also by definition the moment of inertia about this axis times alpha, which is the omega dt.

This is equation number two, this is equation number three-- the situation is getting out of hand-- this is equation number 4. Then omega T, which is spun up, is omega 0 plus alpha times T, and omega 0 is 0 at the moment of touchdown. I have assumed here that a is constant, and that alpha is constant, but I didn't even have to assume that. As you will see, I'm going to eliminate a, T, and alpha, and convince you that the final result is independent of a, T, and alpha.

If the plane pushes harder down on the wheels, the frictional force will be higher, and the acceleration-- the declleration, I should say-- will be larger. The only thing that will happen is that this pure roll situation will occur earlier, but the final result is still the same.

Look at these equations. I first want to combine this one with this one, and what I'm going to do is I'm going to eliminate F. When I eliminate F between this one and this one, I'm going to get R times m times a equals I times alpha. Now I'm going to eliminate alpha by using this equation, and so I'm going to write down omega T equals R times m times a times T divided by I.

This now-- omega T-- according to this equation, is also v center of mass divided by R. I'll just write v for that-- that is simpler-- just as I have dropped a also as the center of mass. This now must become v divided by R. Therefore, at becomes v times I-- v being the final velocity of the plane-- divided by m R squared. This at I'm going to substitute now into this equation, and I think I have done the job.

It was a lot of massaging and a lot of work, but I think it will work: v equals v 0 minus v times I divided by mR squared. I find, then, that v, the final velocity of the plane, equals mR squared times v 0 divided by mR squared plus I. It's a non-intuitive result. What you see is that the larger I is-- the moment of inertia of the wheels-- the smaller v will be, and that I find kind of intuitive.

If I is very large when you are in pure roll, you dump a lot of energy into that wheel. Remember the kinetic energy of a rotating wheel is 1/2 I omega squared, and you have taken it out of the motion of the plane. I like the idea that the very large value of I gives me a low velocity of v. Notice again that there is no frictional force in here-- there's no a in here, there's no alpha in here, and there's no T in here.

As I stressed earlier, the a, the alpha, the T, and the F can change in time. If the frictional force is enormous because the plane pushes hard, the whole process will go very fast, but the outcome will not change. It's not so intuitive.