Rotation and Translation
Challenge Problems

Problem 1: Frictional forces on bicycle wheels

You are riding your bike along a flat country road. What are the directions and relative magnitudes of the frictional forces on the front and rear tires
in the following situations:

a) you are accelerating;

b) you are pedaling along at a steady pace;

c) you are braking. Both the brake and the pedals work on the rear wheel; there is no brake on the front wheel.

Problem 1 Solutions:

Rolling without slipping: $v_r = v_c$

In rolling without slipping , the velocity of the rim at the contact point with the ground, is equal and opposite to the velocity of the center of the wheel. Using the law of addition of velocities, we conclude that the vector sum of the velocities is zero. This is the statement that the observer at rest on the ground sees the contact point on the rim at rest relative to the ground. Thus the friction between the tire and the ground for an ideal wheel is static friction. Recall that the direction of the static friction force depends on the other forces acting on the wheel.

Case a): If the velocity of the center of the wheel is constant, then the static friction is zero. This is what we mean by an ideal wheel. (We shall soon see that there is another type of friction present calling rolling resistance.)

Case b): If the velocity of the center of the wheel is increasing then the direction of the static friction must point backwards to increase the velocity of the rim of the wheel.

Case c): If the velocity of the center of the wheel is decreasing then the direction of the static friction must point forwards to decrease the velocity of the rim of the wheel.

Slipping and rolling: $v_r > v_c$

In slipping and rolling, the velocity of the rim is greater in magnitude and opposite in direction to the velocity of the center of the wheel so the law of addition of velocities show that the vector sum points backward. Thus the observer at rest on the ground sees the contact point moving backwards relative to the ground. Thus there is a sliding friction between the ground and the wheel and this force acts on the wheel in a forward direction. The effect is to increase the velocity of the center of the wheel and to decrease the velocity of the rim of the wheel until they are equal in magnitude. Then the friction force becomes static and the analysis of the direction of this force was given above for the case of rolling without slipping .

Skidding and rolling: $v_r < v_c$

In skidding and rolling, the velocity of the rim is lesser in magnitude and opposite in direction to the velocity of the center of the wheel so the law of addition of velocities shows that the vector sum points forward. Thus the observer at rest on the ground sees the contact point moving forwards relative to the ground. Thus there is a sliding friction between the ground and the wheel and this force acts on the wheel in a backward direction. The effect is to decrease the velocity of the center of the wheel and to increase the velocity of the rim of the wheel until they are equal in magnitude. Then the friction force becomes static and the analysis of the direction of this force was given above for the case of rolling without slipping .

Now let's consider which cases hold for the motions of the bicycle. The normal force on the rear tire is greater than the normal force on the front tire due to the weight distribution and the connection of the gear train to the rear axle. Thus the magnitude of the friction force will be much greater on the rear tire than the front tire.

a) When the bicycle is accelerating, the rear wheel is slipping and rolling. The sliding friction force points forward and it is this force which is accelerating the bicycle. The front wheel will undergo rolling without slipping but since the velocity of the center of the wheel is increasing, the direction of the static friction must point backwards to increase the velocity of the rim of the wheel.

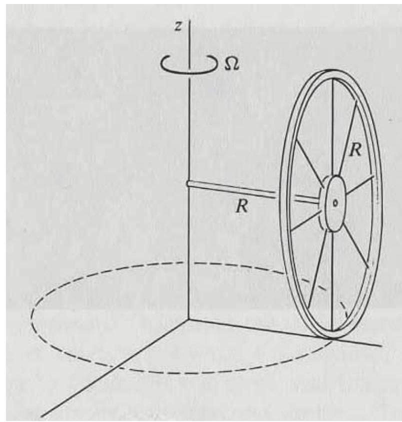
b) **you are pedaling along at a constant rate.** When you are pedaling along at a constant rate, friction must still supply a forward force on the rear wheel to overcome the air resistance which slows down the velocity of the center of the wheel. The wheel is rolling without slipping and the static friction points forward which causes the velocity of the center of the wheel to increase and decreases the velocity of the rim of the wheel to maintain the rolling without slipping condition. The front wheel since it is rolling without slipping at a constant velocity has essentially zero static friction.

c) **you are braking. There is no brake on the front wheel.** When you are braking rear wheel, the rear wheel is skidding and rolling. Therefore there is a sliding friction between the ground and the wheel and this force acts on the wheel in a backward direction. The effect is to decrease the velocity of the center of the wheel and to increase the velocity of the rim of the wheel until they are equal in magnitude. The front wheel is rolling without slipping. Since the velocity of the center of the wheel is decreasing then the direction of the static friction must point forwards to decrease the velocity of the rim of the wheel.

In this analysis we are ignoring the effect of the deformation of the tire which produces a net **rolling resistance** which is always pointing backwards and is a much smaller then the sliding frictions. Depending on the rate of acceleration, it may be comparable to the static friction.

Problem 2:

A thin hoop of mass m and radius R rolls without slipping about the z axis. It is supported by an axle of length R through its center. The hoop circles around the z axis with angular speed Ω. (Note: the moment of inertia of a hoop for an axis along a diameter is $(1/2)mR^2$.)



a) What is the instantaneous angular velocity $\vec{\boldsymbol{\omega}}$ of the hoop? Specify the direction and magnitude.
b) What is the angular momentum $\vec{\mathbf{L}}$ of the hoop about a point where the axle meets the z axis? Is $\vec{\mathbf{L}}$ parallel to $\vec{\boldsymbol{\omega}}$?

Problem 2 Solutions:

a) Because the radius of the hoop and the length of the axle are the same, when the hoop completes one circuit around the circle it also completes one complete revolution about the axle. The result is that the spin angular velocity has the same magnitude as the orbital angular speed, $\omega_{\text{spin}} = \Omega$. Due to this restriction, we cannot neglect the vertical component of angular velocity or angular momentum. The angular velocity of the hoop about its center is $\vec{\boldsymbol{\omega}} = \Omega(\hat{\mathbf{k}} - \hat{\mathbf{r}})$ (note that the horizontal component is directed radially inward in the above figure.

b) About the specified point, there are three contributions to the angular momentum: the horizontal component (often known as the "spin" angular momentum), the motion of the center of the wheel about the central shaft (often known as the "orbital" angular momentum) and the fact that the wheel is also rotating about a vertical axis. The angular momentum is then given by

$$\vec{\mathbf{L}} = \omega_{\text{spin}} mR^2(-\hat{\mathbf{r}}) + \Omega mR^2(\hat{\mathbf{k}}) + \Omega\frac{1}{2}mR^2(\hat{\mathbf{k}}) = \Omega mR^2\left(\frac{3}{2}\hat{\mathbf{k}} - \hat{\mathbf{r}}\right);$$

the angular momentum is not parallel to the angular velocity.