I have a uniform rod on a frictionless horizontal surface. Here is that rod. The rod has mass m, length l, and I hit it with a impulse. The impulse is perpendicular to the direction of the rod J, which is a vector. This is the center of the rod O, which is also the center of mass. I want to know-- I'm going to tell you this distance from O to the impulse is h. This angle is 90 degrees.

I want to know after the impulse what the velocity is of the center of mass-- it will be a translation-- and what the angular velocity is of the rod about the center of mass. It's going to rotate. If I hit here, it will rotate clockwise, and if I hit it here, it will rotate counterclockwise. These are the two questions we have on the table.

Let's first apply Newton's First Law: F equals M sometimes a of the center of mass equals M times dv center of mass dt. First of all, F and a could be time-dependent. The impulse is the integral of F dt from initial time before I hit to a final time after I hit. During that hit, however, F does not have to be constant, but this quantity is J. I take the integral of this equation exactly as I did here, so in this equation, this part becomes M times dv center of mass between the initial time before the hit to the final time just after the hit.

We know that this is J. That's the definition of J. This is the mass times the velocity of the center of mass after the hit-- I call that the final state-- minus the velocity of the center of mass initially before it hit, and this one is 0. You see here that J and the velocity of the center of mass are in the same direction, so the velocity of the center of of mass will be exactly parallel to this J. The magnitude of the velocity at the center of mass-- this is the final one. This is the F here. It is after the hit-- I will simply write for that v center of mass, the magnitude is J divided by M. But if you want to keep the vector notation, that, of course, is fine. So J, the impulse, is the momentum change. This is the momentum after the impulse, and this was the momentum before the impulse, but that happened to be 0.

Let's now calculate the angular velocity. This object is going to rotate when I hit it here. I did that earlier, or maybe not earlier, but I did it a different exercise, in which I chose O as my special chosen point of origin. Today, I will do it differently, only to show you the internal consistency, and to show you that you get exactly the same results for omega.

Last time I chose this point, but today I will choose a point anywhere on the line through J. You could choose this point P, or you could choose this point Q, or any point here on this line. The angular momentum relative to either point P or to point Q is 0 before they hit, but it's also 0 after they hit. The angular momentum is conserved. The reason why the angular momentum is conserved is that dL dt, relative to that-- let's take point Q-- which is the torque relative to point Q, equals 0.

The reason why that is 0 is that this force, which may be variable in time, which acts during a certain amount of time between t initial and t final, that forces goes through point Q for the torque 0, so the angular momentum is conserved as long as you choose a point anywhere on the line PQ. The angular momentum is not conserved relative to any other point. It's not conserved relative to point O, for that matter. It is only conserved relative to any point on the line PQ.

The angular momentum, the total angular momentum relative to that point Q equals 0. It is also the sum of the angular momentum of the center of mass relative to point Q plus the angular momentum about the center of mass, and so the sum of these two must be 0.

Let's call this contribution 1, and let's call this contribution 2. Contribution 1: the angular momentum of the center of mass relative to point Q equals rQ cross P momentum of the center of mass. I'll tell you what rQ is. If this is point Q, then this vector would be r of Q, and so I have a cross product between this vector and the momentum. We have to multiply the center of mass by the mass to give you the momentum.

This value in terms of magnitude equals h times M times v center of mass. I'm sure you can figure that out for yourself that the direction is out of the paper. Even though I have vectors here, and I left the vector notation up here, as is indicated by the symbol, the arrow comes out of the paper.

The contribution of 2 equals the moment of inertia about the center of mass times omega, which is the angular velocity that we want to calculate. I will give it a vector notation. If h is positive, and the object rotates clockwise, then this vector would be in the paper, and its magnitude is 1/12 Ml squared times the angular velocity that we want to calculate.

These are in opposite directions. One is out of the paper, perpendicular to the paper, and one is in the paper, perpendicular to the paper. I can arbitrarily assign a minus sign to this, and a plus sign to this, and so I then get the equation that minus h times M times v center of mass plus 1/12 Ml squared times the angular velocity, which I want to calculate, equals 0. I lose my M, and I know that the velocity of the center of mass-- we calculated that earlier-- equals J divided by M, the magnitude.

You'll find immediately from this equation that the angular velocity-- I will write down a v to remind you that it is the angular velocity about the center of mass-- equals 12 times h times J divided by M times l squared. This is exactly the result that we found before when we didn't choose Q as our point of origin, but when we chose a as our point of origin-- not a, we chose O as a point of origin. The only difference is that I called this distance d in that exercise, whereas here I call it h. I think h is nicer, because when you call this d, you have so many d's floating around, you can be confused.

If you look at this result, you see very clearly that if h is larger than 0, you get a rotation in this direction. It's a clockwise rotation. If h is smaller then 0, you are hitting below O, and then you get a counterclockwise rotation. If h equals 0, then you get that the angular velocity equals 0.

Notice that the larger h is, the larger the angular velocity is in terms of magnitude. Omega, the angular velocity, clearly is a strong function of h. It grows linearly with h. The velocity of the center of mass is completely independent of h, and when you look at it, perhaps that is kind of intuitive. I think it agrees with my intuition.