

Let's now go to our next problem, and our next problem is 10.8.

I have a car, which is mass  $M$ , and it has four wheels. Each wheel has radius  $R$  and moment of inertia  $c$  rotating about the center of axis of symmetry. We have pure roll-- that's a given-- so that there is no slip. That's a given at this moment. The speed is constant.

Here we have that wheel, and it's rotating around with angular velocity  $\omega$ , this is the center  $c$ , this is the radius  $R$ , and there is a velocity  $v$  of the center of mass in situation 1, which is  $\omega R$ . This is one particular moment, so that's why I give it a 1. This is only true because there is pure roll. Otherwise, this would not be true.

What is the angular momentum? The angular momentum about point  $c$ , and I refer to moment 1, that is  $I$ , moment of inertia about point  $c$ , about this axis, times  $\omega$ . So  $L$  at that moment in time, in the beginning, equals the moment of inertia divided by  $v$ , divided by  $R$ . That is not too difficult.

What is the net force on this system? The arrow should be here. That must be 0, because the speed is constant, so there cannot possibly be a net force on this system.

Now we are suddenly going to brake, and we lock the wheels. We do this braking until the velocity of the center of mass has reached the value  $v_2$ , and the question now is, what is the acceleration of the car? Here is this wheel, screeching on the road, because the wheels have been locked. This still has the velocity  $v_1$ , the center of mass has  $v_1$ , and the bottom has  $v_1$ , because I've locked it, and everything is going with  $v_1$ . We would assume to a good approximation that that is true.

Here I will have a frictional force, which is the maximum value possible because the whole thing is skidding. The friction maximum-- and I'll take all four wheels together, because it makes no difference-- is the total mass of the car times  $g$ , which is this normal force. All four wheels together must push back upwards with a force  $Mg$  times the friction coefficient  $\mu_k$ , which is the kinetic friction coefficient because it's sliding, and that must be  $Ma$ . You'll find immediately that the acceleration-- in this case, it's a deceleration-- equals  $\mu_k g$ . The car is experiencing acceleration in the same direction as the frictional force, and it slows down, and if you take  $\mu_k = 0.1$ , then you would get  $a = 0.1g$ , which is a substantial reduction and a substantial deceleration.

Now comes the question, what is the distance that the car travels from the moment  $v_1$  until the moment that it has reached this new velocity  $v_2$ ? Well,  $v$ , in general, equals  $v_0$  plus  $at$ -- this is a general equation, and at the end, when it has reached this velocity-- at  $v$ , I will call this  $v_2$ . At time equals 0, I will call this  $v_1$ , and the acceleration is negative, so I'll put a minus sign in. I know that  $a$  now should be positive, because this minus sign already takes into account that it is in this direction. We've got minus  $a$ , and let's call the time that this has occurred, that this velocity has been reached, let's call that time  $t_2$ .

You'll find immediately that the time that it takes equals  $v_1$  minus  $v_2$  divided by  $\mu_k$  times  $g$ . This is how long it takes for this car under the influence of this squeaking friction to slow down to this velocity.

Now the question is, what is the distance traveled? Well, that is relatively easy now, because  $x$  at that time  $t_2$  minus  $x$  at time  $t_0$ -- a general equation which we had early in the course-- equals  $v_0 t$  plus  $1/2 a t^2$ . This is the moment that we have reached this velocity. If we substitute in the values that we have, this is  $v_1$ -- this should be  $t_2$ , by the way-- times  $t_2$  minus, and I'll put in the minus sign,  $1/2 a t_2^2$ . I put in the minus sign, because now I'm going to put in this equation for a positive sign.

When I calculate-- this is the distance traveled-- I know what  $a$  is, because I've just calculated  $a$ , and so I find that the distance traveled equals  $1/2$  times  $v_1$  squared minus  $v_2$  squared divided by  $\mu_k$  times  $g$ . This is the distance traveled. Notice that if  $\mu_k$  goes up, that the distance traveled goes down. That is quite pleasing, and that is quite intuitive, I would say.

Now I release the brakes. Now we have this object, these wheels with radius  $R$ , and now all the velocity is here. I have  $v_2$ , because I have squeaked the brakes, everything is locked, and it is skidding over the road. I have a frictional force here, which on one wheel only would be  $\mu_k$  times  $m$  over 4, if you take only one wheel into account, and this wheel is now going to torque up. It's going to torque up until you have pure roll, when the  $v$  of the center of mass becomes a new  $\omega$  times  $R$ .

This problem is exactly identical to the one that I had done last week, so I will not do this problem. I'll just remind you that it's not so intuitive that the  $n$ -th time and the  $n$ -th situation, not the  $n$ -th time but the  $n$ -th situation, namely the velocity of the center of mass, and the angular velocity is completely independent of how long it takes for this acceleration to occur. That means how strong the frictional force is and how hard the car pushes down on the road. If the car pushes down on the road, the friction is high, because the normal force is high. It's all independent of that.

You will get one answer for omega, and you get one answer for the center of mass when you're going to use this for all situations, and it's all independent of those quantities, which is by no means so obvious. I did that last time, so I will not do it again. I did it last time in glorious detail, from what I remember.