

I have a yo-yo, and the yo-yo has two disks. This disk has radius R , and this inner disk is in a little portion here, which we assume has no mass-- that's radius little r . This one has mass m , this also has mass m , and these two are identical. The yo-yo is released at 0 speed and it's going to rotate around this axis-- let's call this point C in the middle.

The moment of inertia about this point rotating about this axis is twice the moment of inertia of the disks rotating about our axis of symmetry, and since I have two disks, I must multiply by 2 times $\frac{1}{2} mr^2$, so this is mr^2 . That's the moment of inertia about that axis.

I'm looking at this situation from the side-- this is this inner core of radius r , the rope is wrapped around this inner core, and this is the outside radius capital R . The forces-- we have here $2mg$. Remember, this was mass m , and this was mass m , so we have to double that. Here we have the tension T in this rope.

Clearly we have again $\alpha = a/r$, because we have pure roll, so we can use that wherever we want to. We're going to apply Newton's Law, and this system is going to be accelerated downwards, so $2mg - T = 2ma$ -- that's equation number one.

In the torque situation, I will take the torque relative to point C , which is the moment of inertia about point C times α , and that equals little r times T . I take the moment of inertia relative to point C . There is this perpendicular distance little r times T , and that equals the moment of inertia about C , which was mr^2 times a divided by R , so I have already eliminated α . This is the second equation.

Here we have two equations we have two unknowns, and you should be able to solve for both a and T . It's completely identical to what we have done before-- there really is no difference, except that the geometry is slightly more complicated.

If now you're being asked what the angular velocity is-- ω -- when y equals h . You release this at 0 speed, and you let it just unwind. You want to know what the angular velocity is, or what the center of mass velocity is as y equals h . Now you have a choice out of two: you can either use the value for a as I just used in the previous problem, and you grind your way all the way through, or what I would advise you to do because it's just a little easier, you use the conservation of energy. When you use the conservation of energy, you should get exactly the same answer.

If I used the conservation of energy in my previous problem-- the problem that I was so proud of-- the object is going to move. This is the previous problem where we had simply the object with mass m , and the object is going to move over a distance h . The energy released as mgh -- Massachusetts General Hospital-- that must go into the kinetic energy of the center of mass, which is $\frac{1}{2} m$ center of mass plus $\frac{1}{2} I$ of c times ω squared. Again, this is that previous problem whereby we had I of c equals $km r$ squared, and this was that point c .

Since v equals ωR , you can ride on mgh equals $\frac{1}{2} m \omega$ squared r squared plus $\frac{1}{2} km R$ squared ω squared. You have one equation with one unknown ω , and you find ω is $\frac{1}{R}$ times the square root of $2gh$ divided by $1 + k$. Not to confuse the issue, but this is part of what I plan to do in my problem number one.

Notice that I find exactly the same value-- there's no difference, this is precisely the value I found for ω -- I think it dropped on the floor. We used it earlier by calculating a , massaging it all the way through, but here we use a conservation of energy. I would advise you in the case of this problem that we're dealing with now, which is this problem 10.10, if you want to calculate now with ω is as y equals h or $y v$. The center of mass is y equals h , and I would recommend you use this technique. Of course, $I c$ is now a little different from what we have here, but the concept is the same.

When you use the conservation of energy, it always goes faster than when you grind all the way through and you use your a values. That's not so easy.