It's very general-- I have a rope, and the rope is wrapped around an object, which could be a disk. Let's call this point Q, let's call this point P, and this is the center of the object. The object could be a sphere or a cylinder, and moment of inertia about that point C-- about the axis of rotation perpendicular to the paper-- equals kmR squared. If this were a solid sphere, then k would be 2/5, if it were a solid cylinder or a disk, then it would be 1/2, and if it were a hollow sphere, then k would be [? 2/3 ?] All these combinations you can look up in tables.

The question now is what is the acceleration of the center of mass as it goes down? What is the tension T [UNINTELLIGIBLE] and not to be confused with the period? What is omega as a function of time as the thing starts to unroll? You can add to that, what is the speed of the center of mass as a function of time?

The thing you're going to rotate around with angular velocity omega-- this is clearly a situation of no slip. If I make an enlargement here-- that means if it rotates over an angle d theta, and if the rope never slips here, then the ropes moves over an angle ds. It's immediately obvious that d theta times R, which is the radius, equals ds. Divide left and right by dt and you'll find omega R equals v. This is the tangential velocity of the rope here, but that is also the same as v c because there is no slip.

If you take the derivative of this so that omega dot becomes alpha, then you get alpha times R equals the acceleration of the center of mass. I will repeatedly use today that alpha equals the acceleration off the center of mass divided by R. That is a necessary condition for no slip.

Let's put all the forces in here. Here we have mg, here we have the tension T, and that's all there is on this object. Newton's Law must hold: F equals ma. This holds for all the sources on the system with the acceleration of the center of mass. What we see if we take mg positive down is mg minus T, which is up, must be m times a c.

That is my first equation, and I have two unknowns-- I have T and a as unknowns. Now we get that to the torque relative to point C equals I relative to point C times alpha as the moment of inertia through this [? axel. ?] The torque relative to point C is T times this perpendicular distance R, so I can write down that this is R times T. That is also I C, which we know is kmr squared times alpha, but alpha is a C divided by R. This is my second equation-- I have two equations with two unknowns in which you can solve easily for a, and you can solve easily for T.

I will now leave you with the massaging, but I will give you the answers, because they are kind of interesting: a equals g divided by 1 plus k. The tension equals kmg divided by 1 plus k. What you see here is that the acceleration, independent of the mass, is only a function of geometry-- it only depends on k.

I will jump a surprise on you with something that you may not have known, and I don't blame you if you don't know this. If I have a pure roll situation-- that means if this object has a pure roll, and we discussed earlier what we mean by p pure roll if this is radius R-- it's going around with angular velocity omega, and it has a velocity of the center of mass v c, then for one thing, v C equals omega R. There's something else which is interesting: if I look at the rotation about this point P-- something I will not prove-- then this motion of this object as it goes into pure roll, is also a perfect rotation of this whole object about this point with exactly that same omega. I don't prove it, and maybe you are will to do that, but I will assume that you will take my word for it.

That is the reason, as perhaps you remember where I mentioned earlier, that the velocity here equals 0, and that the velocity here equals 2 v C. If you were to take a position somewhere here, then this is now the R that you have to take, this is the pivot point you have to take, and so v at that location relative to point P, equals omega R. You take the same omega, and so you get a vector in that direction. It's a very complicated motion, but what is interesting is that it is also a pure rotation about this point with angular velocity omega, except that now the distances R depend on where you are on the circumference.

If we take this for granted, there is something that you can do that is very nice: we've taken the torque relative to point C-- why not take the torque relative to point Q, which is also the same as the torque relative to point P? You will see there's no difference, because it is R cross F. Since T goes through P and Q, you only deal with this force, and it is a perpendicular distance. The torque, relative to point P, which is also the torque relative to point Q equals mgR, as I just explained. That, of course, is the moment of inertia relative to that point Q or through that point P times alpha, and for alpha you can write down a divided by R, if you want that.

What is now this moment of inertia? We use the parallel axis theorem. We get mgr equals the moment of inertia about the center of mass, which is kmR squared plus the distance through the [? new ?] axis squared times the mass, so-- plus m R squared. This is simply an application of the parallel axis theorem. I have this times alpha now, and so this becomes times a divided by R.

What you have here-- maybe to your surprise-- one equation with one unknown. There's no T in here, even, and when you work this out, you will find exactly the same that you found before. Maybe you'll find this even easier-- you find a equals g divided by 1 plus k. If you want to calculate what T is, you can go to the previous part of the problem, and you substitute a in the equation for T, and obviously you'll find exactly the same answer.

Let us take some cases whereby k is 2/5, just to give you some feeling for it-- a would be roughly 0.7 g, and the tension would be roughly 0.29 g. This would be a solid sphere. If I take k equals 1/2, which would be a solid cylinder, a would be 2/3 g-- which is very close, by the way-- and T would 1/3 mg. That is also very close to that value.

It doesn't make all much difference to whether you take a solid sphere or whether you take a solid disk. That's interesting all by itself.

Now we start this system-- so we have this object, we drop it from a position y equals 0, and we drop it over a distance h. We now want to know what the angular velocity is when this object reaches this height h. You can call this w for y equals h, or you can also calculate the velocity of the center of mass for y equals h. I'll first do it in the clumsy way, and then I'll do it in a better way.

The motion in this direction is accelerated-- it's constant acceleration in a in the y direction, so y of t equals y 0, which is 0, plus 1/2 at squared. That equals h-- that's the distance that it travels. You find immediately that t equals the square root of 2h divided by a.

What is the velocity of the center of mass at any moment in time? That is at, because it starts at 0 speed. That is a times this t, which is the square root of 2h over a, and so that is the square root of 2 ah. The the speed of the center of mass after this thing has unrolled over a distance h would be the square root of 2 g h divided by 1 plus k. Now we also have the velocity.

What is omega when y equals h? Well, v equals omega R, and omega equals this number divided by R, so I get 1 over R times the square root of 2 g h divided by 1 plus k.

Notice that the velocity that it reaches when it has unrolled over a distance h is again independent of mass. It only has the geometry in k, and this is the angular velocity. Again, the angular velocity only depends on geometry, which is maybe not all that intuitive.