So let's go now through the idea of center of mass. We will spend the rest of the time on this incredible concept of the center of mass.

If I have an object which is not a point, but which has a finite size, or I have many little points which interact with each other, then I define the center off mass as follows: m i times r i-- this is a vector-- equals the total mass of the system times the position vector of the center of mass. What do I mean by this? Let this be o-- this could be my laboratory friend. Here I could have some crazy object-- I carve out here a little element m i, a little volume element which has mass m i.

Here is the position vector r i-- let this be the center of mass. Here, I have a position vector r center of mass. That is the [UNINTELLIGIBLE] of this equation, and m total is this sum of the mass of all these little elements--it's the total mass of this object. You could, if you want to, split this into an x direction and into a y direction. If you prefer, you could write this vector equation-- you could write then m i x i equals the total mass times x of the center of mass. You can do the same for writing here a y, and writing here a y.

The center of mass is very, very special indeed-- it has remarkable qualities and characteristics, and they are not so obvious at all. Let's take a closer look at the center off mass. Let's take a situation whereby-- this is my laboratory, and this is the origin of my laboratory-- I have here lots of particles which interact with each other. They could be a cluster of stars somewhere out in the universe: they interact with each other, they could attract each other, they could repel each other with electric charge, they could collide, they could explode-- anything is allowed. Summon all the external forces on those together, and if the net external force is 0, then the center of mass, which may have a velocity in this direction-- the v center of mass, relative to my laboratory-- then the total momentum can always be written as the total mass of this entire cloud of particles times the velocity of the center of mass.

In the absence of external force, this is conserved, and that means that dp dt equals 0. The velocity of the center of mass would be dr central of mass dt-- that's the definition of velocity. In the absence of an external force-- a net external force-- the momentum of the center of mass is conserved and cannot change, regardless of how these particles interact with each other. If there's no external force, because the dp dt is the net external force, then notice that since F external equals the total mass of all the particles times a of the center of myass-- notice that there's some remarkable property going on here.

The property that you see here-- you take the derivative of this equation, and you get the dp dt, which is F, and you get dp dt, which is a.

The remarkable property is that if there is an external force, that the center off mass behaves as if it were a point particle, which has the total mass of all the little particles together, and acted upon by the sum of the external forces that act together on all these particles together. I have to add them all up, and that is not so obvious at all-- I would refer you to page 5.7 of your book.

Let us return now to a case where we don't have many little particles, but that we have big particle. Since the center of mass acts as a point, and all the mass is in that point, it's clear that when I hang an object-- a crazy object-- on a string, that the center of mass, which acts like a point, will always fall exactly below the vertical line of the string. If I have an object like this, and the center of mass may be somewhere here, I could hang it like this, and then the center of mass-- if I damp out all the oscillations-- must lie exactly on the vertical, somewhere along this vertical line. I don't know where, but it's somewhere on this vertical line-- I now suspend it from here, and the center of mass must again be somewhere along the vertical line, and so where the two intersect-- which is somewhere here-- there is the center of mass. It's a nice way to determine the center of mass of some crazy object-- this is not so crazy, but I can make it crazy.

Suppose I take my bracelets off, and I put my bracelets here on one side-- now it's a crazy object. Now, it's clear that the center of mass is shifting in this direction, and notice this vertical line now goes through here, so somewhere here is the center of mass. If I now suspend it from here, the center of mass must also fall below this line, and where the two intersect, that is where the center of mass is.

I leave you with the determination of centers of mass, and you'll get lots of opportunities in this problem set. The center of mass is very special indeed: look at this equation. Total-- you've seen it before-- times r of the center of mass.

Suppose you are not in the laboratory frame, but you are in a frame of reference which moves with the center of mass-- quite possible, right? It has the same velocity of the center of mass. There are no external forces, and therefore, the center of mass will continue with the same velocity forever and ever. But now you move in a frame of reference with the center of mass-- that means that dr c dt as seen from your center of mass frame must be 0, because the center of mass is not moving in your frame.

That means that the sum of m of i times dr i dt is also 0, but this is also the sum of m of i times v of i-- that is also the sum of individual momenta of all these little individual particles, if I return to particles that interact with each other, and that must be 0. If I move in the center of mass, the frame of reference before after the collision and with external forces, the momentum is always 0. This is very characteristic, and this can come in very handy in solving certain problems.