System of Particles and of Conservation of Momentum
Challenge Problems Solutions

Problem 1 Center of Mass of the Earth-Sun System

The mean distance from the earth to the sun is $r_{e,s} = 1.49 \times 10^{11}\ \text{m}$. The mass of the earth is $m_e = 5.98 \times 10^{24}\ \text{kg}$ and the mass of the sun is $m_s = 1.99 \times 10^{30}\ \text{kg}$. The mean radius of the earth is $r_e = 6.37 \times 10^{6}\ \text{m}$. The mean radius of the sun is $r_s = 6.96 \times 10^{8}\ \text{m}$. Where is the location of the center of mass of the earth-sun system? Is it inside the sun's radius or outside?

Solution:

Choose an origin at the center of the sun and a unit vector $\hat{\mathbf{i}}$ pointing towards the earth, then $\vec{\mathbf{r}}_s = \vec{\mathbf{0}}$. The center of mass of the earth-sun system is given by

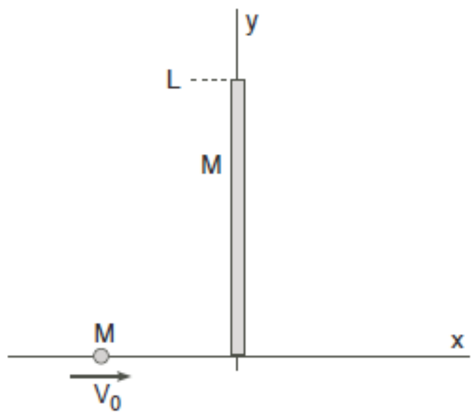
$$\vec{\mathbf{R}}_{cm} = \frac{1}{m_{\text{sys}}} \sum_{i=1}^{i=N} m_i \vec{\mathbf{r}}_i = \frac{1}{m_e + m_s}(m_e \vec{\mathbf{r}}_e + m_s \vec{\mathbf{r}}_s) = \frac{m_e \vec{\mathbf{r}}_e}{m_e + m_s} = \frac{m_e r_{e,s}}{m_e + m_s}\hat{\mathbf{i}} \qquad (0.1)$$

$$\vec{\mathbf{R}}_{cm} = \frac{(5.98 \times 10^{24}\ \text{kg})(1.49 \times 10^{11}\ \text{m})}{(5.98 \times 10^{24}\ \text{kg} + 1.99 \times 10^{30}\ \text{kg})}\hat{\mathbf{i}} = 4.48 \times 10^{5}\ \text{m}\ \hat{\mathbf{i}} \qquad (0.2)$$

The sun's mean radius is $r_s = 6.96 \times 10^{8}\ \text{m}$ so the center of mass of the earth-sun system lies well within the sun.

Problem 2 Center of Mass of the Particle-Rod System

A slender uniform rod of length L and mass M rests along the y-axis on a frictionless, horizontal table. A particle of equal mass M is moving along the x-axis at a speed V_0. At $t = 0$ the particle strikes the end of the rod and sticks to it. Find the position $\vec{\mathbf{R}}_{cm}(t)$ and velocity $\vec{\mathbf{V}}_{cm}(t)$ of the center of mass of the system as a function of time.



Solution: The center of mass of the system of rod and particle is given by the expression

$$\vec{\mathbf{R}}_{cm}(t) = \frac{m_1\,\vec{\mathbf{r}}_1(t) + m_2\,\vec{\mathbf{r}}_2(t)}{m_1 + m_2}.$$

where $\vec{\mathbf{r}}_1(t)$ is the position of the center of mass of the rod and $\vec{\mathbf{r}}_2(t)$ is the position of the particle. The center of mass of the rod at $t = 0$ is given by the expression

$$\vec{\mathbf{r}}_1(t = 0) = (L/2)\hat{\mathbf{j}}.$$

At time $t = 0$, the particle is at the origin so $\vec{\mathbf{r}}_2(t = 0) = 0$. So the center of mass of the system at time $t = 0$ is

$$\vec{\mathbf{R}}_{cm}(t = 0) = \frac{M(L/2)\hat{\mathbf{j}}}{2M} = (L/4)\hat{\mathbf{j}}.$$

The velocity of the center of mass is given by

$$\vec{\mathbf{V}}_{cm}(t) = \frac{m_1\,\vec{\mathbf{v}}_1(t) + m_2\,\vec{\mathbf{v}}_2(t)}{m_1 + m_2}.$$

The rod is at rest at $t = 0$ and so $\vec{\mathbf{v}}_1(t = 0) = \vec{\mathbf{0}}$. At time $t = 0$, the particle is moving in the positive x-direction and the velocity is given by $\vec{\mathbf{v}}_2(t = 0) = V_0\hat{\mathbf{i}}$. So the velocity of the center of mass of the system at time $t = 0$ is given by

$$\vec{\mathbf{V}}_{cm}(t) = \frac{M V_0\hat{\mathbf{i}}}{2M} = (V_0 / 2)\hat{\mathbf{i}}.$$

Because there are no external forces acting on the system, the velocity of the center of mass of the system is constant and the position of the center of mass of the system moves according to

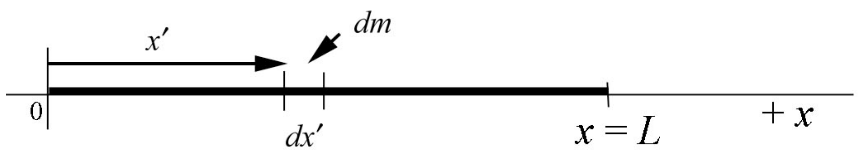
$$\vec{\mathbf{R}}_{cm}(t) = \vec{\mathbf{R}}_{cm}(t = 0) + \vec{\mathbf{V}}_{cm}\, t = (L / 4)\hat{\mathbf{j}} + (V_0 / 2)\, t\, \hat{\mathbf{i}}.$$

Problem 3: Center of Mass of a Rod A thin non-uniform rod has length L and total mass M and the linear mass density varies with the distance x from the left end according to

$$\lambda = \frac{\lambda_0}{L} x$$

where λ_0 is a constant and has SI units $[\mathrm{kg} \cdot \mathrm{m}^{-1}]$. Find λ_0 and the position of the center of mass with respect to the left end of the rod.

Solution: Choose a coordinate system with the rod aligned along the x-axis and origin located at the left end of the rod. Choose an infinitesimal mass element dm located a distance x'. Let the length of the mass element be dx'.



Then

$$dm = \lambda(x')dx' = \lambda = \frac{\lambda_0}{L} x' dx' \,. \qquad \textbf{(0.3)}$$

The total mass is found by integrating the mass element over the length of the rod

$$M = \int_{x'=0}^{x=L} \lambda(x')dx' = \frac{\lambda_0}{L} \int_{x'=0}^{x=L} x'dx' = \frac{\lambda_0}{2L} x'^2 \Big|_{x'=0}^{x'=L} = \frac{\lambda_0}{2L}(L^2 - 0) = \frac{\lambda_0}{2} L \qquad \textbf{(0.4)}$$

Therefore

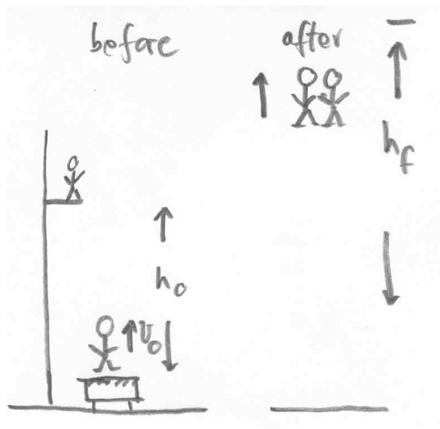
$$\lambda_0 = \frac{2M}{L} \qquad \textbf{(0.5)}$$

The center of mass is again found by integration

$$\begin{aligned} \vec{\mathbf{R}}_{cm} &= \frac{1}{M} \int_{\text{body}} \vec{\mathbf{r}}\, dm = \frac{1}{M} \int_{x'=0}^{x} \lambda(x')x'dx' \,\hat{\mathbf{i}} = \frac{\lambda_0}{ML} \int_{x'=0}^{x} x'^2 dx' \,\hat{\mathbf{i}} = \frac{2}{L^2} \int_{x'=0}^{x} x'^2 dx' \,\hat{\mathbf{i}} \\ \vec{\mathbf{R}}_{cm} &= \frac{2}{3L^2} x'^3 \Big|_{x'=0}^{x'=L} \hat{\mathbf{i}} = \frac{2}{3L^2}(L^3 - 0)\,\hat{\mathbf{i}} = \frac{2}{3} L \,\hat{\mathbf{i}} \end{aligned} \qquad \textbf{(0.6)}$$

Problem 4 Two Acrobats

An acrobat of mass m_A jumps upwards off a trampoline with an initial y-component of the velocity $v_{y,0} \equiv v_0$. At a height h_0, the acrobat grabs a clown of mass m_B. Assume that the time the acrobat takes to grab the clown is negligibly small.



How high do the acrobat and clown rise? How high would the acrobat go if the acrobat and the clown have the same mass?

Solution:

Choose the acrobat and the clown as the system. The first important observation to make is that there is a collision between the acrobat and the clown. This collision is completely inelastic in that the two bodies collide and "stick' together after the collision. The details of the collision are determined by the internal forces in the system. Since this is a one-dimensional motion, let's choose an origin at the trampoline and the positive y-axis upwards.

There are two important states to identify in this problem.

State 1: Immediately before the collision. Acrobat A just arrives at platform located at $y_{1,A} = y_{2,B} = h_0$ with velocity $\vec{\mathbf{v}}_{1,A} = v_{1,A}\hat{\mathbf{j}}$, immediately before grabbing Clown B. Denote this time by t_1.

The collision lasts a time Δt_{col}. During this time interval, acrobat A grabs Clown B.

State 2: Immediately after the collision. At the end of the interval, the two acrobats rise together with velocity $\vec{\mathbf{v}}_2 = v_2\hat{\mathbf{j}}$. Denote the time at the end of this interval by $t_2 = t_1 + \Delta t_{col}$. The key assumption is that the collision time is instantaneous $\Delta t_{col} \cong 0$.

The impulse delivered by the external gravitational force is zero during the collision because the collision was assumed to be instantaneous. Therefore during the collision, the total momentum of the system is constant. If the collision lasts a significant length of time, there would be some slowing down of the acrobat A during the collision. then we need to calculate this effect. However by assuming the collision is instantaneous, we can ignore this slowing down, and therefore the change in the system momentum is zero.

From one-dimensional kinematics, the y-component of the velocity of the acrobat A at the t_1 is given by

$$(v_{1,A})_y = (v_0^2 - 2gh_0)^{1/2} . \tag{0.7}$$

State 1 to State 2:

Momentum in state 1 (immediately before collision) is only due to acrobat A

$$\vec{\mathbf{p}}_{1,A} = m_A (v_{1,A})_y \,\hat{\mathbf{j}} = m_A (v_0^2 - 2gh_0)^{1/2} \,\hat{\mathbf{j}} . \tag{0.8}$$

The momentum in state 2 (immediately after the collision) is

$$\vec{\mathbf{p}}_2 = (m_A + m_B) v_{2,y} \,\hat{\mathbf{j}} . \tag{0.9}$$

Since momentum is unchanged,

$$m_A (v_0^2 - 2gh_0)^{1/2} \,\hat{\mathbf{j}} = (m_A + m_B) v_{2,y} \,\hat{\mathbf{j}} . \tag{0.10}$$

The y-component of the velocity of the acrobat A at the t_2 is given by

$$. \, v_{2,y} = \frac{m_A}{m_A + m_B} (v_0^2 - 2gh_0)^{1/2} \tag{0.11}$$

Again from one-dimensional kinematics, the final height of the acrobat and the clown is given by

$$h_f = \frac{1}{2g} (v_{2,y})^2 + h_0 . \tag{0.12}$$

We can use our above result for the y-component of the velocity immediately after the collision to find the final height in terms of the initial y-component of the velocity of acrobat A and the initial height of clown B,

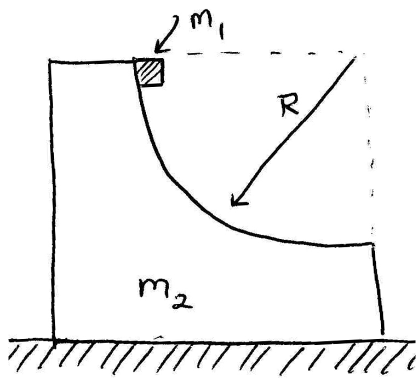
$$h_f = \frac{1}{2g}\frac{m_A{}^2}{(m_A + m_B)^2}(v_0^2 - 2gh_0) + h_0 . \tag{0.13}$$

When the mass of the acrobat is equal to the mass of the clown $m_A = m_B$, the mass ratio becomes $\frac{m_A{}^2}{(m_A + m_B)^2} \cong \frac{1}{4}$ and so the height becomes

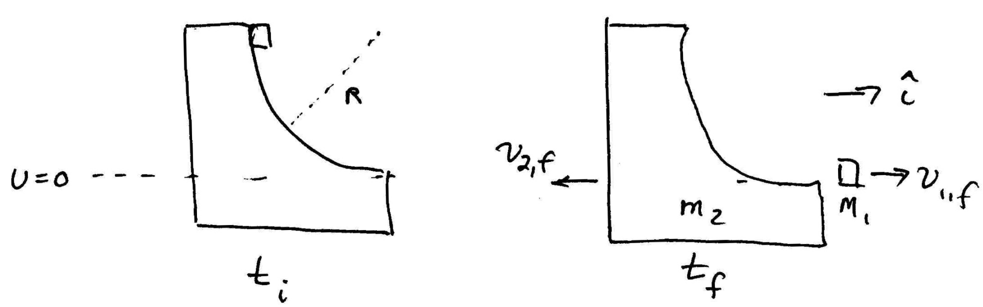
$$h_f = \frac{1}{2g}\frac{m_A{}^2}{(m_A + m_B)^2}(v_0^2 - 2gh_0) + h_0 \cong \frac{1}{8g}(v_0^2 - 2gh_0) + h_0 = \frac{1}{4}(\frac{v_0^2}{2g} + 3h_0) . \tag{0.14}$$

Problem 5 Recoil

A small cube of mass m_1 slides down a circular track of radius R cut into a large block of mass m_2 as shown in the figure below. The large block rests on a table, and both blocks move without friction. The blocks are initially at rest, and m_1 starts from the top of the path. Find the velocity $\vec{\mathbf{v}}_1$ of the cube as it leaves the block.



Solution: If we consider the earth-cube-block system, there are no external forces in the horizontal direction so the horizontal component of momentum is constant, $p_{x,i} = p_{x,f}$. We can ignore the horizontal motion of the earth and so the momentum of the block and cube is constant. Initially the system is at rest, so the final horizontal component of the momentum is zero. Also energy is constant $E_i = E_f$ since there is no external work (the gravitational force is an internal force and the work done is describable by a change in potential energy. The initial and final states are shown in the figure below.



The condition that momentum in the x-direction is constant becomes

$$0 = m_1 v_{1,f} - m_2 v_{2,f} . \tag{0.15}$$

We can solve Eq. (0.15) for the final velocity of the block ,

$$v_{2,f} = \frac{m_1 v_{1,f}}{m_2}. \quad (0.16)$$

The condition that the energy is constant becomes

$$U_i = U_f + K_f. \quad (0.17)$$

If we choose as the zero point for potential energy the height that the cube leaves the block, then $U_f = 0$, and Eq. (0.17) becomes

$$m_1 g R = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2. \quad (0.18)$$

We can now substitute Eq. (0.16) into Eq. (0.18) to find that

$$m_1 g R = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 \left(\frac{m_1 v_{1,f}}{m_2} \right)^2 = \frac{1}{2} m_1 v_{1,f}^2 \left(1 + \frac{m_1}{m_2} \right). \quad (0.19)$$

Thus we can solve Eq. (0.19) for the velocity of the cube

$$\vec{\mathbf{v}}_{1,f} = \sqrt{\frac{2 g R m_2}{(m_2 + m_1)}}\, \hat{\mathbf{i}}. \quad (0.20)$$

Problem 6 People jumping off a flatcar

N people, each of mass m_p, stand on a railway flatcar of mass m_c. They jump off one end of the flatcar with velocity u relative to the car. The car rolls in the opposite direction without friction.

a) What is the final velocity of the car if all the people jump at the same time?

b) What is the final velocity of the car if the people jump off one at a time?

c) Does case a) or b) yield the largest final velocity of the flat car.

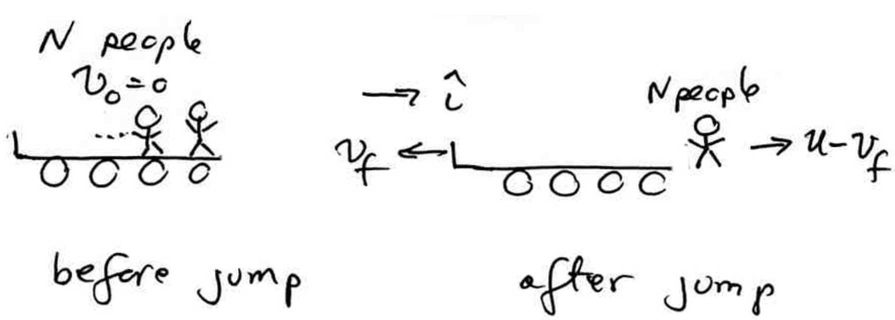
Solution:

We begin by choosing a reference frame at rest with respect to the ground and identify our system as the flatcar and all the people. Since there are no external forces in the horizontal direction, the horizontal component of the momentum of the system is constant

$$p_{x,i} = p_{x,f}. \tag{0.21}$$

We can use this fact to solve for the final speed v_f of the flatcar when all the people jump off together. We need to be careful to use the fact that the speed of each jumper relative to ground is given by $|u - v_f|$.

We take as our initial state the car and people at rest. The final state is immediately after all the people have jumped off. The schematic momentum diagram below shows these states.



Then the initial x-component of the momentum is

$$p_{x,i} = 0. \tag{0.22}$$

The final x-component of the momentum is

$$p_{x,f} = -m_c v_f + Nm_p(u - v_f)\,. \tag{0.23}$$

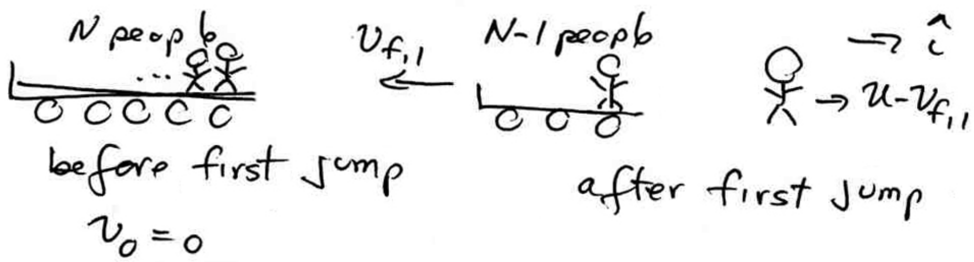
Substituting Eq. (0.22) and Eq. (0.23) into Eq. (0.21) yields

$$0 = -m_c v_f + Nm_p(u - v_f)\,. \tag{0.24}$$

We can solve Eq. (0.24) for the final velocity of the car,

$$v_f = \frac{Nm_p}{Nm_p + m_c}u\,. \tag{0.25}$$

(b) if the people jump off one at a time, we need to be more careful. Again the momentum of the system is constant but we have N jumps.



Before the first jump, the momentum is still zero. Immediately after the first person jumped, the x-component of the momentum is

$$p_{x,f,1} = -((N-1)m_p + m_c)v_{f,1} + m_p(u - v_{f,1})\,. \tag{0.26}$$

Since the x-component of the momentum is constant we have that

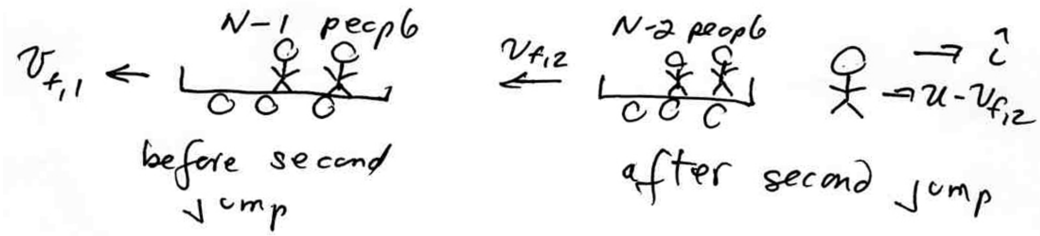
$$0 = -((N-1)m_p + m_c)v_{f,1} + m_p(u - v_{f,1})\,. \tag{0.27}$$

We can solve this equation for the speed of the car after the first jump and find that

$$v_{f,1} = \frac{m_p}{Nm_p + m_c}u\,. \tag{0.28}$$

Note that this is $1/N$ of the speed found when the people all jumped at once (Eq. (0.25).

Now let's consider the second jump.



The x-component of the momentum before the jump is

$$p_{x,i,2} = -((N-1)m_p + m_c)v_{f,1} \tag{0.29}$$

The x-component of the momentum immediately after the second person jumped is

$$p_{x,f,2} = -((N-2)m_p + m_c)v_{f,2} + m_p(u - v_{f,2}) . \tag{0.30}$$

Again applying the fact that the x-component of the momentum is constant yields

$$-((N-1)m_p + m_c)v_{f,1} = -((N-2)m_p + m_c)v_{f,2} + m_p(u - v_{f,2}) . \tag{0.31}$$

We can rewrite this equation as

$$-((N-1)m_p + m_c)v_{f,1} = -((N-1)m_p + m_c)v_{f,2} + m_p u . \tag{0.32}$$

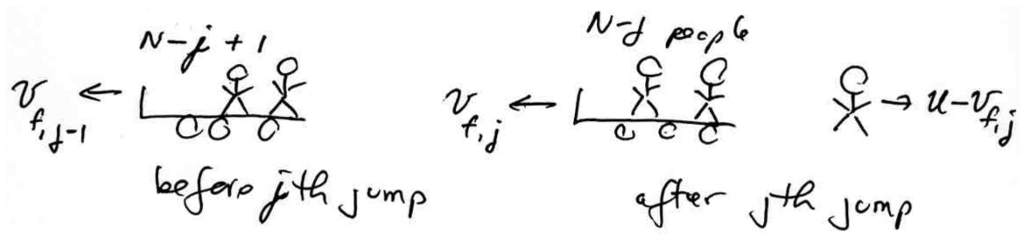
After dividing through by $((N-1)m_p + m_c)$ and rearranging Eq. (0.32) becomes

$$v_{f,2} = v_{f,1} + \frac{m_p}{(N-1)m_p + m_c}u . \tag{0.33}$$

Substituting Eq. (0.28) into Eq. (0.33) yields the speed of the car immediately after the second person jumped off

$$v_{f,2} = \frac{m_p}{Nm_p + m_c}u + \frac{m_p}{(N-1)m_p + m_c}u . \tag{0.34}$$

Notice that the second term on the right hand side of Eq. (0.33) is larger than the first term on the right hand side, so the speed is now larger than $v_{f,2} > 2v_{f,1} = \dfrac{2}{N}v_f$.



By induction, the speed after the jth person jumped off is

$$v_{f,j} = \frac{m_p}{Nm_p + m_c}u + \frac{m_p}{(N-1)m_p + m_c}u + \cdots + \frac{m_p}{(N-(j-1))m_p + m_c}u \,. \quad (0.35)$$

Hence the speed after the last person (the Nth) person jumped off is

$$v_{f,N} = \frac{m_p}{Nm_p + m_c}u + \frac{m_p}{(N-1)m_p + m_c}u + \cdots + \frac{m_p}{m_p + m_c}u \,. \quad (0.36)$$

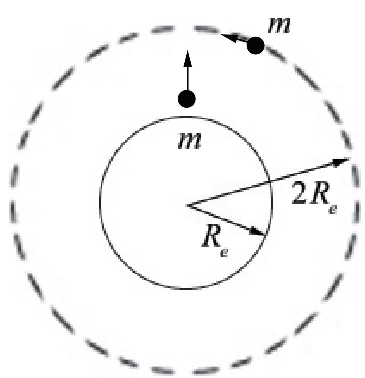
(c) To compare the answers to the previous two parts, note that each term in Eq.(0.36) is larger than the previous one, so we can conclude that

$$v_{f,N} > v_f = \frac{Nm_p}{Nm_p + m_c}u \,. \quad (0.37)$$

Without doing the calculation, we can alternatively use a proof by contradiction to understand why jumping one at a time produces a larger final velocity for the flatcar. Consider case A to be the everybody-jump-at-once case, and case B the one-at-a-time case. Let v_f and $v_{f,N}$ be the final speed of the flatcar in cases A and B, respectively. Then, since each jumper is specified to have a speed u relative to the flatcar's speed immediately after his jump, in case A every jumper ends with an x-component of the velocity $u - v_f$. Now suppose that $v_f > v_{f,N}$. Then each jumper in case B has a final x-component of velocity greater than or equal to $u - v_{f,N}$, and hence larger than the x-component of the jumpers in case A, which is $u - v_f$. Thus the total x-component of the momentum of the jumpers in case B is greater than in case A, so the magnitude of x-component the recoil momentum of the flatcar must also be greater in case B (we need to take the magnitude of the x-component of the recoil momentum because the recoil is in the negative x-direction and so the x-component is negative). Thus we have contradicted our hypothesis. Similarly, if we suppose that $v_f = v_{f,N}$, we could conclude that all the jumpers except the last in case B would have an x-component of momentum larger than each jumper in case A, so again we would have a contradiction. Thus, $v_{f,N} > v_f$ is the only possibility.

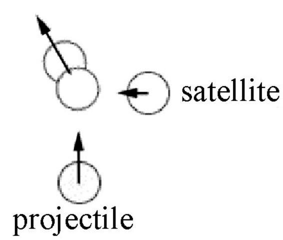
Problem 7 Space Collision

A projectile of mass m is fired vertically from the earth's surface with an initial speed that is equal to the escape velocity. The radius of the earth is R_e, the mass of the earth is M_e, and the universal gravitational constant is G. Express your answers to the questions below in terms of M_e, R_e, m, and G as needed.



a) What is the initial speed of the projectile when it is launched from the surface of the earth?

When the projectile is a distance $2R_e$ from the center of the earth, it collides with a satellite of mass m that is orbiting the earth in a circular orbit. After the collision the two objects stick together. Assume that the collision is instantaneous.



b) What is the speed of the projectile, just before the collision, when it is a distance $2R_e$ from the center of the earth?

c) What is the speed of the satellite, just before the collision, when it is in a circular orbit of radius $2R_e$?

d) What is the speed of projectile and satellite immediately after the collision?

Solution

The escape speed of the projectile occurs when the energy of the projectile-earth system is zero, so conservation of energy can be expressed as

$$\frac{1}{2}m_p v_{p,esc}^2 - \frac{Gm_p m_e}{R_e} = 0$$

We can solve this expression for the escape speed is

$$v_{p,esc} = \sqrt{\frac{2Gm_e}{R_e}}$$

When the satellite reaches a height $r = 2R_e$, the energy is

$$\frac{1}{2}m_p v_p^2 - \frac{Gm_p m_e}{2R_e} = 0\,.$$

Therefore the speed at a height $r = 2R_e$ is

$$v_p = \sqrt{\frac{Gm_e}{R_e}}$$

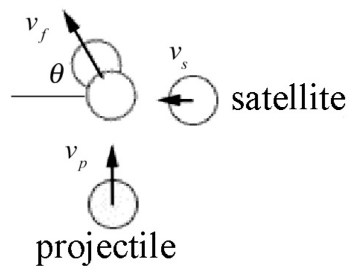
c) Newton's Second Law for the satellite becomes

$$\frac{Gm_s m_e}{(2R_e)^2} = \frac{m_s v_s{}^2}{2R_e}$$

So the velocity of the satellite is

$$v_s = \sqrt{\frac{Gm_e}{2R_e}}$$

a) Momentum is constant during the instantaneous collision. Choose horizontal and vertical directions and θ to be the angle with respect to the horizontal that the combined objects emerge after the collision.



If the masses are equal $m_p = m_s = m$. Then the momentum equations become

$$vertical: mv_p = 2mv_f \sin\theta$$
$$horizontal: mv_s = 2mv_f \cos\theta$$

Square and add these two equations yields

$$m^2(v_s{}^2 + v_p{}^2) = 4m^2 v_f{}^2$$

which we can solve for the final speed

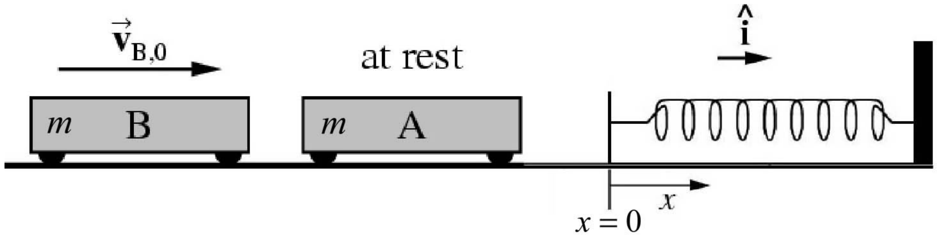
$$v_f = \frac{1}{2}(v_s{}^2 + v_p{}^2)^{1/2}.$$

Substituting for the two speeds yields

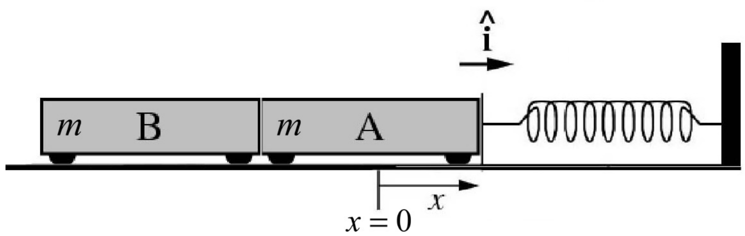
$$v_f = \frac{1}{2}\left(\left(\frac{Gm_e}{2R_e}\right) + \left(\frac{Gm_e}{R_e}\right)\right)^{1/2} = \sqrt{\frac{3Gm_e}{8R_e}}.$$

Problem 8 Spring and Carts

Cart B of mass m is initially moving with speed $v_{B,0}$ to the right, as shown below. It collides and sticks to a second identical cart A of mass m that is initially at rest.



a) What is the speed of the two carts immediately after the collision? Express your answers in terms of m and $v_{B,0}$ as needed.



b) How far does the spring compress when the spring and carts first come to a stop? Express your answers in terms of m, k, and $v_{B,0}$ as needed.

c) How long does it take the right end of cart A to first return to the position $x = 0$? Express your answers in terms of m, k, and $v_{B,0}$ as needed.

d) Set $t = 0$ to be the time immediately after the collision. Write down an expression for the position of the right end of cart A as a function of time for the interval immediately after the collision until the right end of cart A first returns to the position $x = 0$? Express your answers in terms of m, k, and $v_{B,0}$ as needed.

Solution: The system consisting of the two carts has no external forces acting on it so momentum is constant. Therefore

$$m_B v_{B,0} = (m_A + m_B)v_a = 2m_B v_a, \tag{0.38}$$

so

$$v_a = \frac{1}{2} v_{B,0} \tag{0.39}$$

b) There is no external work done on the carts-spring system during compression so the energy is constant, hence

$$\frac{1}{2}(2m)v_a^{\ 2} = \frac{1}{2} k x_{\max}^2 \tag{0.40}$$

Thus

$$x_{\max} = \sqrt{\frac{2m}{k}} v_a = \sqrt{\frac{2m}{k}} \frac{v_{B,0}}{2}. \tag{0.41}$$

c) It takes the block half a period to return to the position $x = 0$. The angular frequency is

$$\omega = \sqrt{\frac{k}{2m}} \tag{0.42}$$

The period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2m}{k}} \tag{0.43}$$

So the block returns to $x = 0$ at time

$$t_1 = \frac{T}{2} = \frac{\pi}{\omega} = \pi\sqrt{\frac{2m}{k}} \tag{0.44}$$

d) The equation for the position is generally

$$x(t) = C\cos(\omega t) + D\sin(\omega t) \tag{0.45}$$

At $t = 0$,

$$x(t = 0) = 0 = C. \tag{0.46}$$

The x-component of the velocity is

$$v_x(t) = \frac{dx(t)}{dt} = -\omega C\sin(\omega t) + \omega D\cos(\omega t) \tag{0.47}$$

At $t = 0$,

$$v_x(t=0) = v_a = \frac{1}{2} v_{B,0} = \omega D\,. \tag{0.48}$$

Therefore substituting Eq. (0.42) into Eq. (0.48) and solving for D yields

$$D = \frac{1}{2\omega} v_{B,0} = \frac{1}{2}\sqrt{\frac{2m}{k}} v_{B,0}\,. \tag{0.49}$$

So substituting Eq. (0.46) and Eq. (0.49) into Eq. (0.45) yields

$$x(t) = \frac{1}{2}\sqrt{\frac{2m}{k}} v_{B,0} \sin\left(\sqrt{\frac{k}{2m}} t\right) \tag{0.50}$$