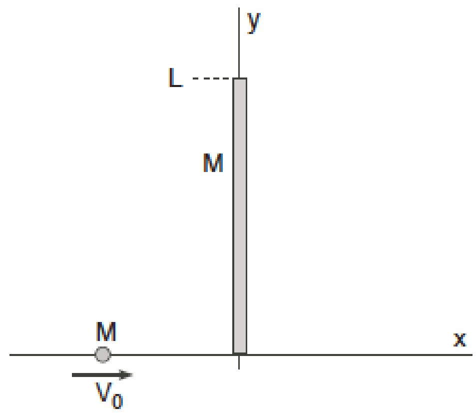
# System of Particles and of Conservation of Momentum
## Challenge Problems

### Problem 1 Center of Mass of the Earth-Sun System

The mean distance from the earth to the sun is $r_{e,s} = 1.49 \times 10^{11}\ \text{m}$. The mass of the earth is $m_e = 5.98 \times 10^{24}\ \text{kg}$ and the mass of the sun is $m_s = 1.99 \times 10^{30}\ \text{kg}$. The mean radius of the earth is $r_e = 6.37 \times 10^{6}\ \text{m}$. The mean radius of the sun is $r_s = 6.96 \times 10^{8}\ \text{m}$. Where is the location of the center of mass of the earth-sun system? Is it inside the sun's radius or outside?

**Problem 2 Center of Mass of the Particle-Rod System**

A slender uniform rod of length $L$ and mass $M$ rests along the y-axis on a frictionless, horizontal table. A particle of equal mass $M$ is moving along the x-axis at a speed $V_0$. At $t = 0$ the particle strikes the end of the rod and sticks to it. Find the position $\vec{\mathbf{R}}_{cm}(t)$ and velocity $\vec{\mathbf{V}}_{cm}(t)$ of the center of mass of the system as a function of time.



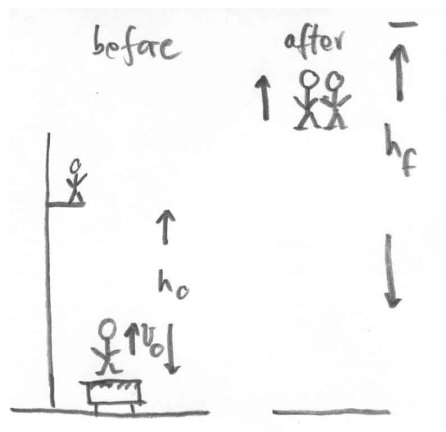
**Problem 3: Center of Mass of a Rod** A thin non-uniform rod has length $L$ and total mass $M$ and the linear mass density varies with the distance $x$ from the left end according to

$$\lambda = \frac{\lambda_0}{L} x$$

where $\lambda_0$ is a constant and has SI units $[\mathrm{kg} \cdot \mathrm{m}^{-1}]$. Find $\lambda_0$ and the position of the center of mass with respect to the left end of the rod.

**Problem 4 Two Acrobats**

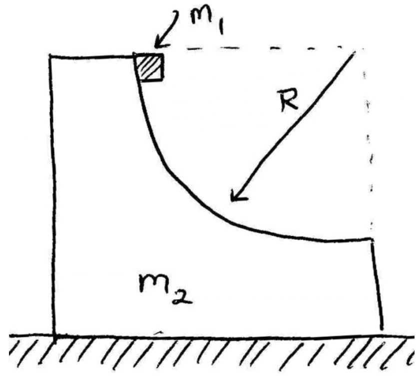
An acrobat of mass $m_A$ jumps upwards off a trampoline with an initial y-component of the velocity $v_{y,0} \equiv v_0$. At a height $h_0$, the acrobat grabs a clown of mass $m_B$. Assume that the time the acrobat takes to grab the clown is negligibly small.




How high do the acrobat and clown rise? How high would the acrobat go if the acrobat and the clown have the same mass?

**Problem 5 Recoil**

A small cube of mass $m_1$ slides down a circular track of radius $R$ cut into a large block of mass $m_2$ as shown in the figure below. The large block rests on a table, and both blocks move without friction. The blocks are initially at rest, and $m_1$ starts from the top of the path. Find the velocity $\vec{\mathbf{v}}_1$ of the cube as it leaves the block.



**Problem 6 People jumping off a flatcar**

$N$ people, each of mass $m_p$, stand on a railway flatcar of mass $m_c$. They jump off one end of the flatcar with velocity $u$ relative to the car. The car rolls in the opposite direction without friction.

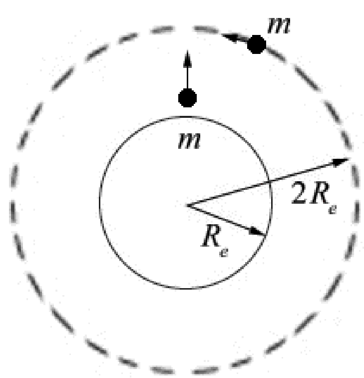
a) What is the final velocity of the car if all the people jump at the same time?

b) What is the final velocity of the car if the people jump off one at a time?

c) Does case a) or b) yield the largest final velocity of the flat car.

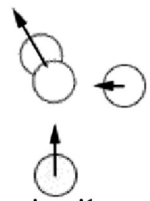
**Problem 7 Space Collision**

A projectile of mass $m$ is fired vertically from the earth's surface with an initial speed that is equal to the escape velocity. The radius of the earth is $R_e$, the mass of the earth is $M_e$, and the universal gravitational constant is $G$. Express your answers to the questions below in terms of $M_e$, $R_e$, $m$, and $G$ as needed.




a) What is the initial speed of the projectile when it is launched from the surface of the earth?

When the projectile is a distance $2R_e$ from the center of the earth, it collides with a satellite of mass $m$ that is orbiting the earth in a circular orbit. After the collision the two objects stick together. Assume that the collision is instantaneous.


satellite
projectile

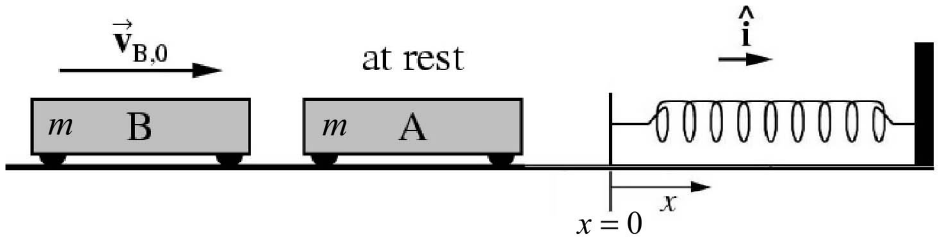
b) What is the speed of the projectile, just before the collision, when it is a distance $2R_e$ from the center of the earth?

c) What is the speed of the satellite, just before the collision, when it is in a circular orbit of radius $2R_e$?

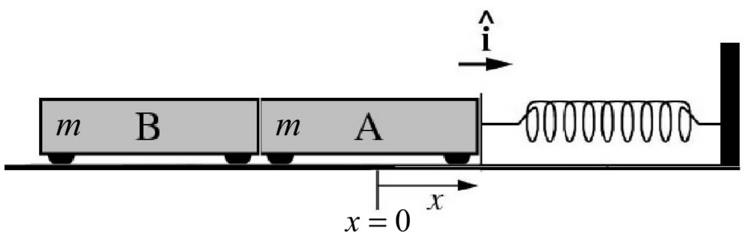
d) What is the speed of projectile and satellite immediately after the collision?

**Problem 8 Spring and Carts**

Cart B of mass $m$ is initially moving with speed $v_{B,0}$ to the right, as shown below. It collides and sticks to a second identical cart A of mass $m$ that is initially at rest.




a) What is the speed of the two carts immediately after the collision? Express your answers in terms of $m$ and $v_{B,0}$ as needed.




b) How far does the spring compress when the spring and carts first come to a stop? Express your answers in terms of $m$, $k$, and $v_{B,0}$ as needed.

c) How long does it take the right end of cart A to first return to the position $x = 0$? Express your answers in terms of $m$, $k$, and $v_{B,0}$ as needed.

d) Set $t = 0$ to be the time immediately after the collision. Write down an expression for the position of the right end of cart A as a function of time for the interval immediately after the collision until the right end of cart A first returns to the position $x = 0$? Express your answers in terms of $m$, $k$, and $v_{B,0}$ as needed.