Momentum and Impulse
Challenge Problems

Problem 1: Estimation

Estimate the magnitude of the momentum of the following moving objects: a) a person walking; b) a car moving along a highway; c) a truck moving along a highway; d) a passenger jet at cruising speed; e) a freight train transporting coal.
Indicate your choice of reference frame for each case.

Problem 2: Impulse and Center of Mass Velocity: ***Elevator Ride***

A person is riding the Building 16 elevator, while standing on the force platform that was used in the lecture demonstration on Monday Oct 16. The force platform measures the force of the person's feet on the platform. The data in Figure 16.1a shows a plot of the force of platform on the person (normal force) vs. time for the person in the elevator.

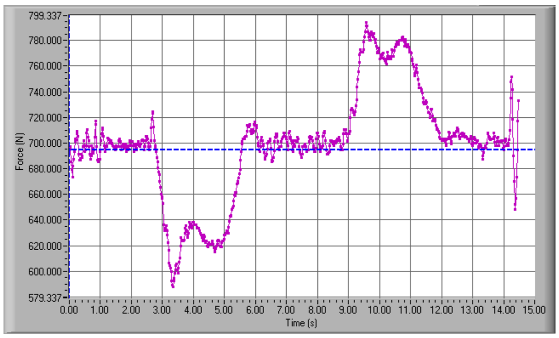


Figure 16.1a

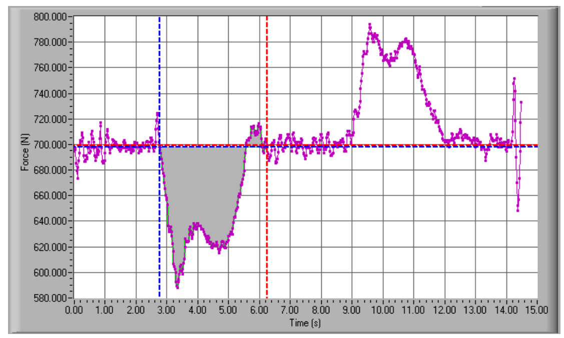
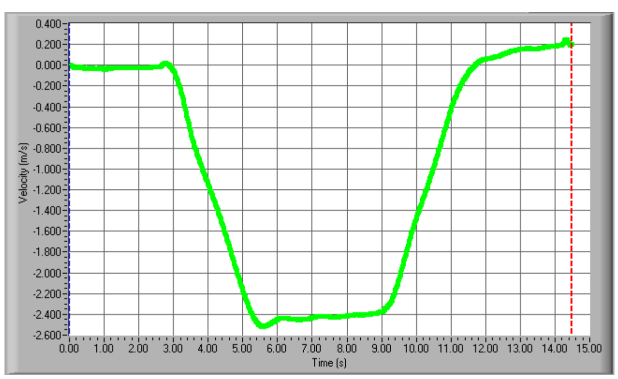


Figure 16.1b

Figure 16.1b highlights the area for the time interval $[t_1 = 2.75\text{ s}, t_2 = 6.35\text{ s}]$. The impulse for this interval is:

$$\vec{\mathbf{I}}[t_1,t_2] = \int_{t_1=2.75\text{ s}}^{t_2=6.35\text{ s}} \vec{\mathbf{F}}^{\text{total}}(t)\, dt = -174\text{ N}\cdot\text{s} \qquad (2.1)$$

Figure 16.2 show a plot of the vertical component of the center of mass velocity of the person vs. time.



a) What does the negative sign signify in Eq.(2.1) in terms of the choice of positive direction for a relevant coordinate axis? Is the choice of positive direction up or down? Based on the plot in Figure 16.1a, describe the motion of the elevator during the interval $[t_1 = 2.75\text{ s}, t_f = 12.0\text{ s}]$. In particular, identify any distinct stages. For each distinct stage describe whether the elevator is moving up or down, whether or not the acceleration is constant (or zero), or non-constant. Explain your reasoning.

b) Explain why the shaded area in Figure 16.1b is with respect to a baseline force of $700\ \text{N}$ and not $0\ \text{N}$.

c) Explain how you can use the data in Figure 16.1a to calculate the change in the vertical component of the velocity of the center of mass of the person? In particular, what is the vertical component of the velocity of the center of mass of the person at (i) $t_1 = 2.75\ \text{s}$, (ii) $t_2 = 6.35\ \text{s}$. How does your calculation compare with the results for the vertical component of the velocity of the center of mass of the person shown in the graph in Figure 16.2?

d) What do you expect the impulse to equal for the time interval $[t_1 = 2.75\ \text{s}, t_f = 12.0\ \text{s}]$?

Problem 3: ***Compressive Strength of Bones***

The compressive force per area necessary to break the tibia in the lower leg is about $F / A = 1.6 \times 10^8 \text{ N} \cdot \text{m}^{-2}$. The smallest cross sectional area of the tibia, about 3.2 cm^2, is slightly above the ankle. Suppose a person of mass $m = 60 \text{ kg}$ jumps to the ground from a height $h_0 = 2.0 \text{ m}$ and absorbs the shock of hitting the ground by bending the knees. **Assume that there is constant deceleration during the collision**. During the collision, the person lowers his center of mass by an amount $\Delta d = 1.0 \text{ cm}$.

a) What is the collision time Δt_{col}?

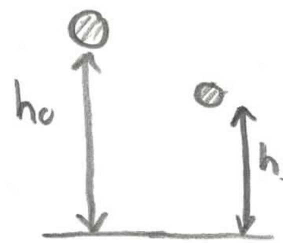
b) Find the average force of the ground on the person during the collision.

c) What is the ratio of average force of the ground on the person to the gravitational force on the person? Can we effectively ignore the gravitational force during the collision?

d) Will the person break his ankle?

Problem 4: ***Momentum and Impulse***

A superball of m_1, starting at rest, is dropped from a height h_0 above the ground and bounces back up to a height of h_f. The collision with the ground occurs over a time interval Δt_c.



a) What is the momentum of the ball immediately before the collision?

b) What is the momentum of the ball immediately after the collision?

c) What is the average force of the ground on the ball?

d) What impulse is imparted to the ball?

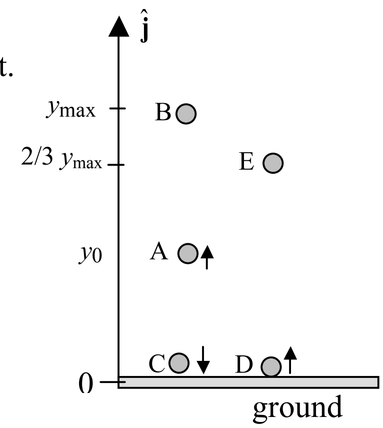
e) What is the change in the kinetic energy during the collision?

Problem 5: Impulse and Newton's Second Law: ***More Bouncing Balls***

A superball of mass m is thrown straight up into the air starting from a height y_0 above the ground. The ball reaches a maximum height y_{max} above the ground. After the ball hits the ground, it bounces back up to a new maximum height equal to $(2/3)y_{max}$. Assume the collision with ground takes place over a time $\Delta t_{collision}$.

The above process is represented in the figure to the right. The important states of the ball are labeled:

A—starting point of the ball:
B – maximum height of the ball:
C – when the ball just hits the ground (moving down):
D – when the ball just bounces from the ground (moving up):
E – new maximum height of the ball.



a) What was the magnitude of the initial velocity v_0 of the ball when it was released?

b) In terms of y_{max} and the gravitational acceleration g, what was the change in the kinetic energy of the ball due to the collision of the ball with the ground?

c) What impulse did the ground impart to the ball?

d) What was the average force of the ground on the ball during the collision?