Momentum and the Flow of Mass
Concept Questions

Question 1 Suppose rain falls vertically into an open cart rolling along a straight horizontal track with negligible friction. As a result of the accumulating water, the speed of the cart

1. increases.
2. does not change.
3. decreases.
4. not sure.
5. not enough information is given to decide.

Answer 3. The water, because it falls vertically, does not change the cart's horizontal momentum. The mass of the cart increases, however, and so its speed decreases.

Question 2

Suppose you drop paperclips ("drop" is taken to mean that the clips are released with no horizontal component of velocity) into an open cart rolling along a straight horizontal track with negligible friction. As a result of the accumulating paper clips, does

1) the kinetic energy and magnitude of the momentum of the cart increase.
2) the kinetic energy and magnitude of the momentum of the cart decrease.
3) the kinetic energy and magnitude of the momentum of the cart stay the same.
4) the kinetic energy increase and the magnitude of the momentum stay the same.
5) the kinetic energy stay the same and the magnitude of the momentum stay increase.
6) the kinetic energy decrease and the magnitude of the momentum stay the same.
7) the kinetic energy stay the same and the magnitude of the momentum stay decrease.
8) the kinetic energy decrease and the magnitude of the momentum increase.
9) the kinetic energy increase and the magnitude of the momentum decrease.

Solution:

The paper clips, in colliding with the cart, change both the horizontal and vertical components of each clip's momentum. The cart must continue to move along the horizontal track, so the net vertical force on the cart is zero, even though the cart (or some of the accumulated clips in the cart) exerts an upward force on the clips, and the cart's vertical component of momentum remains zero.

The clips acquire a forward component of momentum; hence the cart must exert a forward force on the clips. (Or, the accumulated clips exert a forward force on the falling clips, and the cart exerts a forward force on the accumulated clips.) Thus, the clips exert a backward force on the cart, and the magnitude of the cart's momentum decreases. The cart must then lose speed, and hence the cart's kinetic energy will decrease.

Here's a subtle point: If, from Newton's Third Law, the force that a clip exerts on the cart is equal in magnitude but opposite in direction to the force that the cart exerts on the same clip, why don't the products of force and distance cancel?

Second Solution:

The paper clips have zero momentum in the horizontal direction. They all have momentum in the vertical direction. When they collide and come to rest with the cart, they only add mass to the car, so the horizontal momentum of the cart and paperclips diminishes because the mass increases. The vertical momentum of the paper clips is lost during the collision.

The collision is completely inelastic so the cart and paper clips lose kinetic energy as a result of the collision.

Question 3 If a rocket in gravity-free outer space has the same thrust at all times, is its acceleration

1. constant?

2. Increasing?

3. decreasing?

Solution 1. The rocket equation in gravity free outer-space is given by

$$\frac{dm_{f,out}}{dt}u = m_r(t)\frac{dv_r}{dt}$$

The expression on the left-hand side is called the thrust. If the thrust is constant, then the acceleration is increasing because the mass of the rocket is decreasing as the fuel is burned.

Question 4 When a rocket accelerates in a gravitational field, will it reach a greater final velocity if the fuel burn time is

1. as fast as possible?

2. as slow as possible?

3. The final speed is independent of the fuel burn time?

4. I'm not sure.

Solution 1. Suppose a rocket starts from rest in a constant gravitational field. Then after burning all its fuel, the final speed of a rocket is given by

$$v_r(t_f) = u \ln\left(\frac{m_0}{m_{r,0}}\right) - g t_f = u \ln R - g t_f$$

where $m_{r,0}$ is the dry mass of the rocket and m_0 is the dry mass of the rocket plus the mass of the fuel, and u is the speed that the fuel is ejected relative to the rocket. The first term on the right hand side is independent of the burn time. However the second term depends on the burn time, and the shorter the burn time, the smaller the third turn and hence the larger the speed. So the engine should burn the fuel as fast as possible.