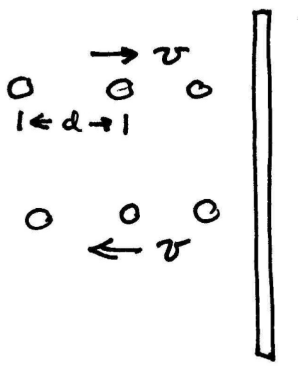
Momentum and the Flow of Mass
Challenge Problems

Problem 1: Stream Bouncing off Wall

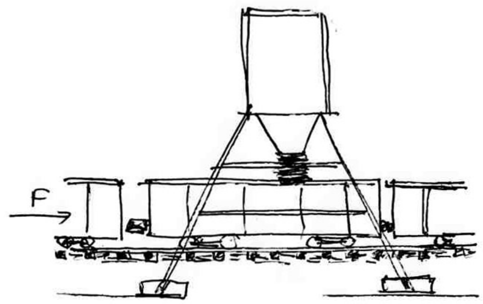
A stream of particles of mass m and separation d hits a perpendicular surface with speed v. The stream rebounds along the original line of motion with the same speed. The mass per unit length of the incident stream is $\lambda = m/d$. What is the magnitude of the force on the surface?



Problem 2 A rocket has a dry mass (empty of fuel) $m_{r,0} = 2\times10^{7}\,\text{kg}$, and initially carries fuel with mass $m_{f,0} = 5\times10^{7}\,\text{kg}$. The fuel is ejected at a speed $u = 2.0\times10^{3}\,\text{m}\cdot\text{s}^{-1}$ relative to the rocket. What is the final speed of the rocket after all the fuel has burned?

Problem 3: Coal Car

An empty coal car of mass m_0 starts from rest under an applied force of magnitude F. At the same time coal begins to run into the car at a steady rate b from a coal hopper at rest along the track. Find the speed when a mass m_c of coal has been transferred.



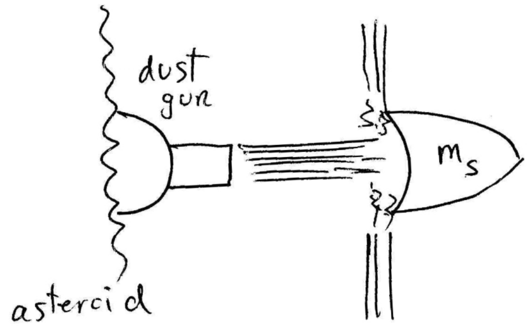
Problem 4: Emptying a Freight Car

A freight car of mass m_c contains a mass of sand m_s. At $t = 0$ a constant horizontal force of magnitude F is applied in the direction of rolling and at the same time a port in the bottom is opened to let the sand flow out at the constant rate $b = dm_s / dt$. Find the speed of the freight car when all the sand is gone. Assume that the freight car is at rest at $t = 0$.

Problem 5: Falling Chain

A chain of mass m and length l is suspended vertically with its lowest end touching a scale. The chain is released and fall onto the scale. What is the reading of the scale when a length of chain, y, has fallen? (Neglect the size of the individual links.) Let g denote the gravitational constant.

Problem 6 A spacecraft is launched from an asteroid by being bombarded by a stream of rock dust. The stream of dust is ejected from the dust gun at a constant rate $dm_e / dt = b$ at a speed u with respect to the asteroid, which we take to be stationary. Assume that the dust comes momentarily to rest at the spacecraft and then slips away sideways; the effect is to keep the spacecraft's mass m_s constant.



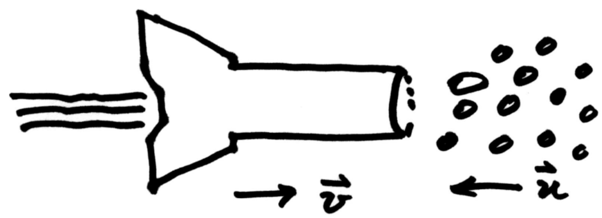
a) Derive an equation for the acceleration dv_s / dt of the spacecraft at time t, in terms of the rate that the dust mass hits the surface of the spacecraft dm_d / dt, the speed of the dust relative to the asteroid u, the mass of the spacecraft m_s, and the velocity of the spacecraft v_s. Show your momentum flow diagrams at time t and time $t + \Delta t$. Clearly identify your system and label all the objects in your system. What is the terminal velocity of the spacecraft? Hint: $dm_d / dt \neq b$.

b) Using conservation of mass, at time t, find an expression for the rate that the dust mass hits the spacecraft, dm_d / dt, as a function of the speed of the spacecraft v_s, the rate that the dust mass is shot from the asteroid $dm_e / dt = b$, and the speed u of the dust relative to the asteroid. Hint: $dm_d / dt \neq b$.

c) Use your results from part b) in part a) to find the speed $v_s(t)$ of the spacecraft as a function of time, assuming $v_s(t = 0) = 0$. (If you get an integral that you are not sure how to integrate, you can leave your answer in integral form.)

Problem 7 Space Junk

A spacecraft of cross-sectional area A, proceeding along the positive x-direction, enters an asteroid storm at time $t=0$, in which the mean mass density of the asteroid storm is ρ and the average asteroid velocity is $\vec{\mathbf{u}}=-u\,\hat{\mathbf{i}}$ in the negative x-direction. As the spacecraft proceeds through the storm, all of the asteroids that hit the spacecraft stick to it.



a) Suppose that at time t the velocity of the spacecraft is $\vec{\mathbf{v}}=v\,\hat{\mathbf{i}}$ in the positive x-direction, and its mass is m. Further, suppose that in an interval Δt, the mass of the spacecraft increases by an amount Δm. Given that there are no external forces, using conservation of momentum find an equation for the change of the spacecraft velocity Δv, in terms of Δm, u, and v?

b) When the spacecraft enters the asteroid storm, the magnitude of its velocity and mass are v_0 and m_0, respectively. Integrate your differential equation in part a) to find the velocity v of the spacecraft when the mass is m.

c) Find an expression for the mass of the asteroids Δm that sticks to the spacecraft within the time interval Δt? (Hint: consider the volume of asteroids swept up by the spacecraft in time Δt).

d) When the spacecraft enters the asteroid storm, the magnitude of its velocity and mass are v_0 and m_0, respectively. What is the mass of the spacecraft at time t? (Use your results from parts c) and b).)

Problem 8 Continuous Mass Transport: falling raindrop A raindrop of initial mass m_0 starts falling from rest under the influence of gravity. Assume that the raindrop gains mass from the cloud at a rate proportional to the momentum of the raindrop, $dm_r / dt = km_r v_r$, where m_r is the instantaneous mass of the raindrop, v_r is the instantaneous velocity of the raindrop, and k is a constant with units $[\mathrm{m}^{-1}]$. You may neglect air resistance.

a) Derive a differential equation for the velocity of the raindrop.

b) Show that the speed of the drop eventually becomes effectively constant and give an expression for the terminal speed.

Problem 9: Rocket Problem A rocket ascends from rest in a uniform gravitational field by ejecting exhaust with constant speed u relative to the rocket. Assume that the rate at which mass is expelled is given by $dm_f / dt = \gamma m_r$, where m_r is the instantaneous mass of the rocket and γ is a constant. The rocket is retarded by air resistance with a force $F = bm_r v_r$ proportional to the instantaneous momentum of the rocket where b is a constant and v_r is the speed of the rocket. Find the speed of the rocket as a function of time.