This is 5 C 6. I believe it's our last problem, or it will the last one that I will do-- I'm not sure, though.

I have two pucks at m 1 and m 2, and they have velocity v 1 and v 2 relative to the center of mass. It's very important: it's not relative to my lab, it's relative to the center of mass. Here is the center of mass, and I am moving with the center of mass frame of reference. I have one particle, m 1, coming in like this with velocity v 1, and I have a particle like this coming in with v 1. Here is my mass m 1, and here is my mass m 2.

When you're working in the frame of reference of the center of mass, think about if the particles always must both come to you, if you have two, or they must both go away from you, or they must be stuck at the center of mass-- that's also possible. That is absolutely necessary, otherwise the net momentum could not be 0-- think about that. This is one such situation. Of course, the center of mass itself could be moving relative to my laboratory frame-- this is what you will see in the reference frame of the center of mass. What you will see in the laboratory frame is something entirely different, and I will get back to that at the end of this problem.

There is a collision, and after the collision, the momentum must again be 0 relative to the center of mass. Let's assume that after the collision, that m 2 moves out with velocity v 2 prime, and m 1 moves out with v 1 prime. As I said, they must either go away from the center of mass, or they must go to the center of mass, or they must be stuck at the center of mass: there's no other possibility, because the momentum relative to the center of mass frame of reference is always 0. I can write down now that m 1 v 1-- it is a momentum vector-- plus m 2 v 2 must be 0, and this is also m 1 v 1 prime plus m 2 v 2 prime. This is the conservation of momentum, but it's 0.

I can introduce a sign convention-- I've done that before-- and I call this plus, I call this negative. Let me call this plus, and let me call this negative. When I do that, I can rewrite this as m 1 v 1 minus m 2 v 2-- that is 0, and that is m 1 v 1 prime minus m 2 v 2 prime. Remember when you saw this that v 2 is larger than 0: it is the speed, but I already know that it is coming in like that. Keep that in mind: all these numbers will be positive, but the direction is nonnegotiable, the direction of this one will be in this direction, and the direction of this one will be in this direction.

Can I solve for v 1 prime and v 2 prime? No, you can't: the best way you can see that is that if v 1 prime equals v 2 prime equals 0, that it would completely satisfy this equation. And why not-- why would it not be possible that when these two objects hit each other like two pieces of putty that they get stuck together right at the center of mass, and that's it, and they stay there? That's certainly one possible solution. There's no way that you can solve for v 1 prime and v 2 prime, unless you know more, and you do know more.

You're being told that the collision is also elastic-- that the kinetic energy is conserved. If kinetic energy is conserved, then you have another equation at your disposal, and you can write that down: you have 1/2 m 1 v 1 squared plus 1/2 m 2 v 2 squared equals, and you write down the kinetic energy after the collision. If now you solve the combination of conservation of kinetic entry with this equation, which is momentum conservation, then you will find after a little bit of massaging, that the magnitude of v 1 prime is 1, and the magnitude or v 2 prime equals v 2. In other words, the speeds have not changed, but the directions have changed, and you have no information about those directions.

Does it mean that nature has a free choice about the directions? Well, not quite: suppose you have two marbles that approach each other, and hit each other. As you can see, it is important to know exactly how they hit-- we call that the impact parameter. If they hit a little bit not exactly at the center, then you get a different direction that they move away from each other, and if they hit head on-- that information we are not being given, and so I will not discuss that any further.

Now I will come back to the question, what do I see in the frame of reference of my laboratory? For that, I have to add now the velocity of the center of mass in the laboratory. Let's assume that the velocity of the center of mass in my laboratory is like this: all these velocities have to be added to this one, to the center of mass, and to this one.

What do I see from my laboratory? I see the vectorial sum of these two. I see this particle flying in this direction relative to [UNINTELLIGIBLE] laboratory, and I would see this particle flying in this direction relative to my laboratory, and they will collide right here. That's all I will see: they will collide here, and by that time, the center of mass has exactly reached that point.

What will I see after the collision? I have to add exactly that same velocity vector of the center of mass-- that velocity vector has not changed, because there are no external forces. I have to add this vector, and this velocity vector, to all these three points, and so what do I see? This is the velocity after the collision of particle 2, and this is the velocity after the collision of particle number 2. What has happened is they came together like this-- bingo-- they hit somewhere here, and then they do that.

That's what you see in your laboratory frame-- they come together, and do this. However, in your center of mass frame of reference, they come together like this, and then afterwards all of a sudden, you see this-- or they stick together. It's very interesting, and we very often use the center of mass in order to shorten a particular problem.

The collision is elastic, so therefore kinetic energy is conserved in the frame of reference of my laboratory, but I haven't used the conservation of kinetic energy as seen from my frame of reference of the center of mass. Is that allowed? Did I not cheat?

No, I didn't. Convince yourself that if kinetic energy is conserved relative to my laboratory, that it's also conserved relative to the center of mass. Keep in mind that there are no external forces on these two objects, so the center of mass relative to my laboratory has a constant velocity, and will always keep that constant velocity. That will very quickly teach you answer why what I did is totally allowed.