Let's now get to problem number 5B.3, and I will get back to the first problem later.

It is a one dimensional collision. I started out with object mass m 1, it has velocity v 1, and it collides with mass m 2, which has velocity v 2. This is before the collision, and here we see the situation after the collision. There is [UNINTELLIGIBLE], m 1, v 1 prime, m 2, and v 2 prime. If there is no external force on the system, only internal, momentum is conserved. The conversation of momentum: m 1 v 1 plus m 2 v 2 equals m 1 v 1 prime plus m 2 v 2 prime. That is equation number one, the conservation of momentum.

I could do delete these bars since I'm dealing with a one dimensional situation, keeping in mind that if the velocities in this direction would then, for instance, assign it a plus sign, but if the velocity is in this direction, it would then have a minus sign. I can delete the bars, not forgetting that this is now my convention, so think of the bars as not being there.

It is given, or I'm telling you that this is also an elastic collision, so you also have that the kinetic energy is conserved: 1/2 m 1 v 1 squared plus 1/2 m 2 v 2 squared equals 1/2 m 1 v 1 prime squared plus 1/2 m 2 v 2 prime squared. That is equation number two, and that is the conservation of kinetic energy, which only holds if this is an elastic collision. This is the conservation of momentum, which in this one big case, observing our sign convention, can then even delete the bars.

If I massage this a little further-- and I will let you do that, since we have little time today, so you will have to do some massaging of this-- you can then show that v 1 minus v 2 equals v 2 prime minus v 1 prime. That is quite remarkable: this is the relative velocity before the collision between the two objects, and this is the relative velocity after the collision between the two objects. It's not so obvious that that would be the same. Of course, it's only true if kinetic energy is conserved-- but what is not required is that m 1 is m 2. That is not required: when you derive that, you will see that that is not a must.

Let's take an example. For instance, I have m 1 equals 5 kilograms, and I have m 2 that equals 22 kilograms. This object is v 1, magnitude 3, and this object is v 2 in the opposite direction with the magnitude of 6 meters per second. Be very careful and think of our sign convention: v 1 minus v 2, and this is now positive 3, plus 3. v 2, itself, is negative-- minus 6, so minus minus 6 becomes plus six, so this is plus 9 meters per second. This now also has to be v 2 prime minus v 1 prime, observing the same sign conventions. This, of course, is quite remarkable when you come to think about it.

You may say, great-- I know the difference in velocity, but what is now v 2 prime, and what is now v 1 prime? Now you have to go back to your equation number one and equation of number two. If you know v 1, you know m 1, you know v 2, and you know m 2, then you have in principle two equations with two unknowns, and you should be able to solve for v 1 prime and v 2 prime. It takes a little bit of hassling, a little bit of massaging, a little bit of algebra: that you can find in books, or you can work on it yourself. It would be a waste of your time if I used my precious 57 minutes right now, so I'll give you the answer.

If you do it correctly, you'll find then that v one prime equals m 1 minus m 2 divided m one plus m 2 times v 1 plus 2 m 2 divided by m 1 plus m 2 times the v 2. Then we have v 2 prime equals 2 m 1 divided by m 1 m 2 times v 1 plus m 2 minus m 1 divided by m 1 plus m 2 times v 2. You can check whether I did that correctly-- if you're changing the first equation with the 1 to a 2 and the 2 to a 1, you obviously should get this equation, because the problem is completely symmetric. Let's take this term, and the one becomes a 2, the 2 becomes a 1, and indeed this looks good. This 2 becomes a 1 here, so I think we are great-- this v 1 is v 2, and this v 2 is v 1, so this looks good.

Remember, this is only true if it is elastic. Remember, that a positive sign means going to the right for the velocity, and a negative sign means going to the left.

Now let's take a very special case-- let's take the case that m 1 equals m 2. Here is m 1 with velocity v 1, here is m 2 with velocity v 2, and if you now take m 1 equals m 2, this is gone, and this is gone. This is 1, this is 1, and what do you find to your great surprise? That v 1 prime equals v 2, and v 2 prime equals v 1.

What does this mean? It means that after the collision, the velocity of 1 is now the velocity of 2, and the velocity of 2 is now the velocity of 1-- that is not so obvious. They exchange their velocity-- it's by no means intuitively so obvious. If we take a very special case that v 2 equals 0, it would follow then from what I just told you, that v 1 prime would be 0-- they exchange, and that v 2 prime would be v 1.

I would like to demonstrate this to you, and the best way I could do that is to have two pucks of equal mass: this one bangs into this one, and this one will be standing still. There has to be no friction, and I could show you, then, after the impact that this one gets this velocity, and this one stands still-- but I don't have a frictionless horizontal table, so I can't do that.

You may say, do it with marbles: if I do it with marbles, there is another problem. I can take two marbles of equal mass, but when I do this collision process on my pad, then they begin to roll. Rolling adds a complication that we have not at all discussed, and that is very special. We can't do it with rolling marbles-- that's a little tricky, and if you slide the marbles, then there is friction. If there is friction, there's external forces-- so let's not do it with marbles either.

I will do it with a pendulum. I have here a pendulum, and I have here a pendulum of equal length and equal mass. I will swing this one towards this one. You will say, there are external forces-- there's gravity, which is very true. There is a gravitational force here, mg, and this accelerates this object, so clearly-- if there's an external force, how can possibly momentum be conserved?

At the moment that they collide, in the x direction, there is no external force. They collide, there are internal forces, and action equals minus reaction, but that's OK-- there are no external forces in the x direction. As they collide in this direction, I have in the x direction-- in the horizontal direction-- an ideal situation: kinetic energy is conserved, because these metal balls are very flexible. They bounce back very nicely, and in addition, I have no external force, so momentum is conserved. So when this object hits this one, this one should stand still, and this one should pick up this speed.

This is something that I want to-- see, I will do it with this famous device, which is called a Newton's Cradle. You see here the two pendulums of equal mass, and we will show it to you from above: that's easier, because I can also zero in then, and then you see it better. In fact, when I zero in, you don't even see the vertical motion, you only see the horizontal motion, which makes it even more convincing.

I move this one out, and at the moment of impact, I have kinetic energy conserved, and I have momentum conserved in the x direction-- there we go. Watch as this hits-- this one will stand still, and this one will pick up that speed. Are you ready-- there we go.

You see? It stands still-- I go back, and it stands still. Of course, if you do this, then you get this alternation, which you have seen, no doubt, many times. A great toy-- can I have the overhead, because I want to be able to adjust the screen? A great toy, and it makes this point very convincing.

Let's now take a situation that m 1 is not m 2, but let's for simplicity still take v 2 equals 0, so our equations becomes a little simple. It's the same equation that I wrote before, but I substitute v 2 equals

0, so this is v 1, and v 2 equals 2 m 1, divided by m 1 plus m 2 times v 1. It's the same equations as before, but I substitute in v 2 equals 0. Let's assume v 1 is in this directions, which we have called positive-- of course, this holds only if kinetic energy is conserved, because that's the way we derive them. It holds only if there is no external force on the system, so the momentum is also conserved-- that's very important.

Notice that v 1 has-- that v 1 prime-- has three options: it can be larger than 0, if m 1 is larger than m 2. It can be equal to 0, if 1 equals m 2-- we saw that before, that's not new-- but it can also be smaller than 0 if m 1 is smaller than m 2. This is intuitively pleasing, because what it means is if a small, low mass ball, like a ping pong ball collides with a baseball, or with a billiard ball, which may be a better example-- then it can bounce back. It's intuitively pleasing, and you see that. If m 1 is smaller than m 2, it will bounce back and will go in this direction.

However, v 2 prime is always larger then 0. Notice no matter how you choose here-- m 1, m 2, or v 1-- if v 1 is the positive direction, then v 2 prime is also in a positive election. That is also very intuitive, because if I have here an object, and I hit it, obviously it must go in this direction. It would be absurd if I hit this object, that it would go back. That would be absurd.

It's completely consistent with what we find here: v 1 prime, you have three possibilities. The v 2 prime always goes into the direction of v 1, and that is exactly what you expect.

Now we'll take an extreme case that m 2 goes to infinity, and so we have a wall. Obviously, if v 2 is 0, the wall is not moving-- let's assume the wall is not moving. If the wall were moving, the kinetic energy of the wall would be infinitely large, and that would be a bit of a problem-- so v 2 better be 0. I find it if I collide an object with m 1-- it goes with the velocity v 1 I'm against the wall-- I find that v 1 prime equals minus v 1. I find that v 2 prime equals 0-- kinetic energy is conserved, and momentum is conserved.

This is quite obvious, you would say: this is the wall, and an object comes in here with a certain velocity, and it bounces back with exactly the same speed, but it reverses its velocity. Yes, this is very intuitive, but I want you to think about something. The kinetic energy is all in here: it's not in the wall. The kinetic energy of the wall, before and after, equals 0. The kinetic energy here is 1/2 m 1 v 1 squared, and it is also 1/2 m 1 v 1 squared when it bounces back, because look-- so that's fine.

Let's now look at the situation of the momentum, and I'll give you some numbers, because it's easier to deal with-- m 1, let's say, is 1 kilogram. That's the ball that bounces off the wall. It comes in with 5 meters per second, and it bounces back with five meters per second. In other words, p 1 has decreased by 10 kilogram meters per second. It's first in the positive direction, and now it's in the negative direction, so it has decreased by 10.

However, I've stated that momentum must be conserved. How can it be-- momentum is changing. Is it perhaps true that the momentum of the wall is also changing, so that the net momentum of the wall plus the ball is 0? It better be, but it's kind of puzzling, isn't it, because the wall has infinite mass, and it has no speed or velocity for the impact or after the impact. You think about that, and it's very important that you get the answer to that, because it's very fundamental.

Conservation of momentum must hold: so yes, indeed, the wall must have changed its momentum, in spite of the fact that it never moved. Isn't that cute? Think about it.