Collisions
Concept Questions

Question 1 Cart A is at rest. An identical cart B, moving to the right, collides inelastically with cart A. They stick together. After the collision, which of the following is true?

1. Carts A and B are both at rest.

2. Carts A and B moves to the right with speed greater than Cart B's original speed.

3. Carts A and B move to the right with a speed less than cart B's original speed.

4. Cart B stops and cart A moves to the right with speed equal to the original speed of cart B.

Answer 3: From conservation of momentum, $mv_{B,0} = 2mv_f$. So $v_f = v_{B,0}/2$. Thus they move away with speed less one half the original speed of cart B.

Question 2 Cart A is at rest. An identical cart B, moving to the right, collides elastically with cart A. After the collision, which of the following is true?

1. Carts A and B are both at rest.

2. Cart B stops and cart A moves to the right with speed equal to the original speed of cart B.

3. Cart A remains at rest and cart B bounces back with speed equal to its original speed.

4. Cart A moves to the right with a speed slightly less than the original speed of cart B and cart B moves to the right with a very small speed.

Answer: 2. Since there are no external forces, the momentum is constant and therefore

$$mv_{B,0} = mv_{B,f} + mv_{A,f},$$

or

$$v_{B,0} = v_{B,f} + v_{A,f}. \tag{2.1}$$

The square of the initial speed is

$$v_{B,0}^2 = v_{B,f}^2 + v_{A,f}^2 + 2v_{B,f}v_{A,f} \tag{2.2}$$

Note that when squaring the initial speed a cross term appears $2v_{B,f}v_{A,f}$. The collision is elastic so conservation of energy implies

$$\frac{1}{2}mv_{B,0}^2 = \frac{1}{2}mv_{B,f}^2 + \frac{1}{2}mv_{A,f}^2,$$

or

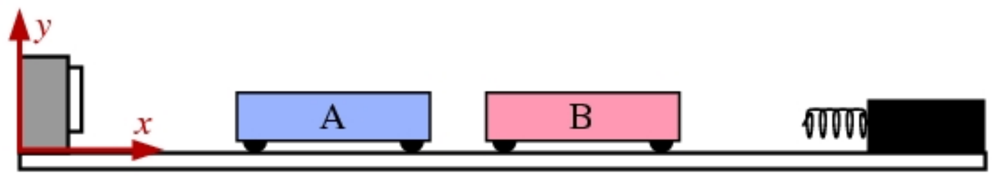
$$v_{B,0}^2 = v_{B,f}^2 + v_{A,f}^2. \tag{2.3}$$

Comparing Eq. (2.2) and Eq. (2.3) implies that the cross term in Eq. (2.2) must vanish. Thus $2v_{B,f}v_{A,f} = 0$. There are two possibilities: Cart B stops $v_{B,f} = 0$ and hence by Eq. (2.1) cart A moves with the initial speed of cart B $v_{B,0} = v_{A,f}$. The other possibility that $v_{A,f} = 0$ and hence $v_{B,0} = v_{B,f}$ just reproduces the initial conditions.

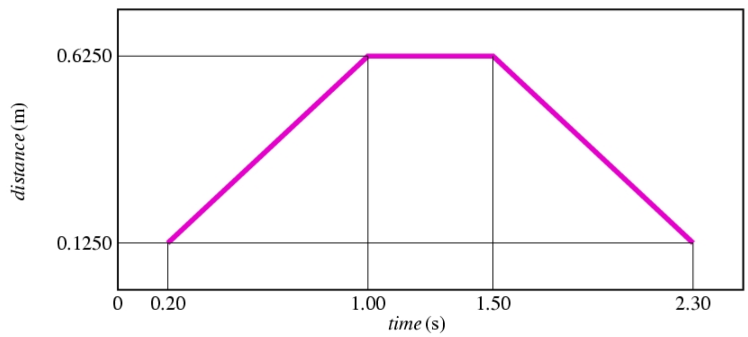
Second argument: Center of mass reference frame:

In the center of mass reference frame both carts approach each other with half the original speed of Cart B in the lab frame. They bounce elastically with the same speeds. So in the lab frame after the collision, Cart B is at rest and Cart A moves to the right with speed equal to the original speed of cart B.

Question 3 The figure below shows an experimental setup to study the collision between two carts.



In the experiment cart A rolls to the right on the level track, away from the motion sensor at the left end of the track. The graph below shows the distance from the motion sensor to cart A as a function of time.



What objects collide when $t = 1.5$ s?

1. Cart B and the spring.
2. Cart B and the motion sensor.
3. Carts A and B.
4. Cart A and the spring.
5. Cart A and the motion sensor.

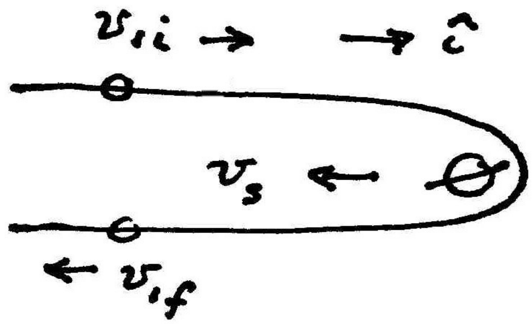
Solution 3. During the time interval from $t = 0.2$ s to $t = 1.0$ s Cart A is moving to the right. At $t = 1.0$ s, cart A and Cart B collide, cart A comes to a complete stop, and cart B moves tot eh right. During the time interval from $t = 1.0$ s to $t = 1.5$ s, cart B moves to the right collides with the spring and reverses direction. At $t = 1.5$ s the two carts collide again. Cart A moves to the left with the same speed as it started with, and therefore Cart B comes to a stop.

Question 4 Two balls that are dropped from a height h_i above the ground, one on top of the other. Ball 1 is on top and has mass m_1, and ball 2 is underneath and has mass m_2 with $m_2 >> m_1$. Ball 2 first collides with the ground and rebounds with speed v_0 Then, as ball 2`starts to move upward, it collides elastically with the ball 1 which is still moving downwards also with speed v_0. The final relative speeds after ball 1 and ball 2 collide is

1. Zero
2. v_0
3. $2v_0$
4. $3v_0$
5. None of the above.

Answer 3. The relative speed between two objects in an elastic collision does not change . before the collision the relative speed was $2v_0$. Therefore after the collision the relative speed is also $2v_0$.

Question 5 A spacecraft with speed v_{1i} approaches Saturn which is moving in the opposite direction with a speed v_s. After interacting gravitationally with Saturn, the spacecraft swings around Saturn and heads off in the opposite direction it approached. The final speed of the spacecraft v_{1f} after it is far enough away from Saturn to be nearly free of Saturn's gravitational pull is



1. v_{1i}
2. v_s
3. $2v_{1i}$
4. $2v_s$
5. $v_{1i} + v_s$
6. $v_{1i} - v_s$
7. $v_{1i} + 2v_s$
8. $v_{1i} - 2v_s$
9. $2v_{1i} + v_s$
10. $2v_{1i} - v_s$

Answer 7. We can think of the fly-by as an elastic collision because energy is constant. Hence the relative speed between the satellite and Saturn does not change. The relative speed as the satellite approaches Saturn is $v_{1i} + v_s$. The relative speed after the satellite is far away from Saturn is $v_{1f} - v_s$. Because the relative speeds are equal $v_{1i} + v_s = v_{1f} - v_s$, therefore $v_{1f} = v_{1i} + 2v_s$

Question 6

An explosion splits an object initially at rest into two pieces of unequal mass. Which piece has the greater kinetic energy? Explain your answer.

1. The more massive piece.
2. The less massive piece.
3. They both have the same kinetic energy.
4. There is not enough information to tell.

Solution: The pieces have the same magnitude of momentum. The kinetic energy of an object of mass m and momentum magnitude p is $p^2/2m$, so the piece with the smaller mass has the larger kinetic energy. (This assumes that the pieces are not rotating, but that's not a consideration in this problem.)