PROFESSOR: There are other ways that you can describe a vector using three numbers in three-dimensional space in which you do not use Cartesian coordinates. And I'll give you an example of that.

Let this be x, y, and z coordinate system. And let it assume we have a vector-- let's take a velocity vector v.

We can identify the angle between the velocity vector and the plus z-axis as theta. So theta would be one number that we could use to express this vector v. If I project this vector v onto the xy plane, which this is the projection. I could then identify another angle measured from the positive x-direction in the direction to the positive y-direction, and I could call this angle phi. That's specifies the direction in this horizontal plane of the projection of v on xy.

We know now the direction in which this vector goes, but we still don't know the magnitude. So if now we also specify the magnitude, we would be in business. So if you knew the magnitude, if you knew theta, and if you knew phi, then you can obviously reconstruct, if you wanted that, the value for vy, you can reconstruct the value for vx and you can reconstruct the value for v of z.

Important is that you realize that this determines uniquely this vector.

How would you go from these numbers if you had these vector v of x, v of y, and v of z to the Cartesian coordinates?

Well, this angle is 90 degrees. So you will see immediately that v of z divided by the magnitude of this vector, the lengths of this vector for which I will simply write v. It's just easier for me now. Divided by v equals the cosine or theta. So v of z equals v times the cosine of theta. That's easy.

If I call this projection of this vector v onto the xy plane, if I call that vxy, then given the fact that this angle is 90 degrees, v of x divided by vxy equals the cosine of phi. And v of y divided by v of xy would be the sine of phi. And if we combine this, we should not have any difficulties to show that the x-component of the velocity equals v times the sine of theta times the cosine of phi. v of y, the component in the y-direction equals v sine theta times the sine of phi. And v of z, the component in the z-direction equals v cosine phi. So if you know theta, you know v, and you know phi, you can find easily all these three components.

Equally well, if I gave you the vector notation in Cartesian coordinates, we could then also go back, of course, to the magnitude to theta and to phi. So let's do an example of that.

Suppose I have a vector A, which is 3x roof minus 2y roof plus 4 z roof. And we would like to find the magnitude of A for which I could write simply A. We would want theta and we would want phi. Let me try to make a drawing. It is not so easy because of this minus 2. If I'm not careful this drawing could become very confusing.

So we have plus 4 in the z-direction and we have plus 3 in the x-direction. Let me try to make you see this three dimensionally. Perhaps not to scale. There we go. OK, I didn't do so badly.

So this point now would be minus 2 on the y-axis and the vector that we are describing, A, would be this vector. It has an x-component plus 3, a z-component plus 4, and a y-component minus 2. And the question now is what is this angle theta? And, what is the angle phi?

Now to find the angle phi, I first have to project the vector A onto the xy plane. So I call this A of xy. And remember how we define the direction of phi.

If this were a view from above and if this is y and this is x, and this is somewhere where the projection of the A vector onto the xy plane is, then the angle phi is defined from the positive x-axis in the direction of the positive y-axis, all the way to here. So this is phi.

So if I draw it here, it's not so easy. This will be phi. And our task now is to determine the magnitude, theta, and phi.

Well, the magnitude is the easiest. A magnitude is the square root of 3 squared, which is 9. Minus 2 squared, which is 4. Plus 16. So that is the square root of 29. What is theta? The cosine of theta equals A of z divided by A. A of z is 4. A is the square root of 29. And so I find that theta equals 42.0 degrees if I round it off. So this angle is about 42 degrees.

Now, how now do we find phi? Well, as we just derived, A of x equals A times the sine of theta times the cosine of phi. And so the cosine of phi equals A of x divided by A sine theta.

Now, we know A of x and we know A. And we know sine theta. So I found this to be 0.83. We want to check that.

And that leaves me then with two choices for phi. I find plus 33.7 degrees and I find plus 326.3 degrees, which, of course, is the same as minus 33.7 degrees. It's just a matter of taste. It is clear that this is the wrong one. Because you all ready see that the angle phi is clearly larger than 180 degrees. I made you this geometrical representation. You can clearly see it. So clearly, this is the winner. If you prefer, however, to write for that minus 33.7, be my guest. What you will then say, well, I really called this angle negative. That's fine. I have no problems with that. So this is phi or this if you so prefer.

If you wanted to find phi using the equation A of y equals A times the sine of theta times the sine of phi, you could do that, too, of course. And you better get the same result. The sine of phi would then be minus 2, which is A of y, divided by A itself, which is the square root of 29, divided by the sine of theta. You know theta. And when I do that I find for phi two possibilities. As always, I find 326.3, which, of course, is the same as minus 33.7. But I also find 213.7 degrees. This one is obviously wrong. And so you see, you have to do a little bit of thinking, and you finally then decide that you get a consistent solution from both this equation as well as from this equation. And there's no question that in our case, the angle of phi is 326 or minus 33.7 if you so prefer.