

MITOCW | MIT8_01SCF10mod07_01_300k

All right, we go to 2.C.3. A tightrope walker. Here is the tightrope. The length of the rope is L and the rope is sagged over a distance y . And here is the tightroper and she holds a very long horizontal bar in her hands for stability.

Now the tightroper has a mass m , so the gravitational pull down is mg . There's no acceleration in this direction, so therefore the rope must push back with a force, which I will call N upwards and the magnitude of N must be mg . Where is this force upwards coming from?

It can only come from the string. There is nothing else. It can only come from the tension in this rope. Now let me simplify the rope a little bit. This is the position where the walker stands, and let the angle here be θ . Let this angle also be θ . And so there is a tension in this rope pulling to the right. T stands for tension. And here there is the same force to the left, T stands for tension. And what is the net force of these two?

The horizontal components cancel. The vertical components is this force plus this force. And the net force vertical N equals $2T$ times the sine of θ .

On any given point of the rope, the tension pulls both to the right and it pulls to the left with equal strength. If that were not the case, there would be a net force at that location of the string and a net force would give it an acceleration. In fact, if the point had 0 mass, it would give it an infinite acceleration. So the force, the tension to the right and the tension to the left must always be exactly the same.

Suppose I cut here this rope. And at the location of the cut, I put in a spring. So here would be then the rope, and here would be the spring, and here would be the rope. Well it should be immediately obvious to you that under the tension, this spring is stretched. It is longer than it wants to be. So the tension makes it longer. And the tension is pulling by the same force to the right as it pulls to the left. And if this spring has a spring constant k , then the magnitude of T would be kx if this spring were extended over the distance x . So effectively, the tension here plays the role that my hands were playing earlier when I played with this body builder accessory and I was doing this.

So always keep in mind that the tension, whenever you deal with tension in a rope or a string or a spring, it is both to the left and it is to the right.

Time check. On the button.

Now, we'll continue with this problem, mg equals $2T \sin \theta$. And in case that y is much, much smaller than L , the sine of θ is approximately y divided by $1/2 L$.

So if θ becomes 0, notice that the tension goes to infinity because this is a given.

Well, this of course, is impossible. It would mean that θ is 0, that y is 0. And y is 0 would mean there is no sag. That the rope would be exactly horizontal, and you know from experience, although you may not be a tightrope walker, but you do know from your experience that because of gravity, when anything pushes down on a rope, the rope has to sag a little. So θ equals 0 is just impossible.

And you'll also notice that when θ becomes larger, if you give it more sag, that T , the tension, will become smaller. And that is also quite intuitive. And then you have a specific example that the book wants you to work out. It gives you a certain value of L , of y . It gives you a value of $\sin \theta$, which I found to be approximately 4 times 10 to the minus 3, which gives you a θ of about 0.2 degrees. About 12 arc minutes. And that then leads to a certain value of the tension applying that equation. And you will see that the tension is horrendous.

Well it is so horrendous because this angle of θ is awfully small, and the mass was 55 kilograms.

Try your intuition on this and I think you will agree with the ideas that the smaller θ , the larger the tension will have to be.