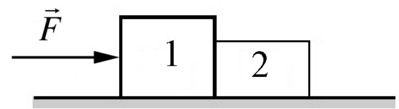
Concept of Force
Challenge Problem Solutions

Problem 1:

Force Applied to Two Blocks Two blocks sitting on a frictionless table are pushed from the left by a horizontal force $\vec{F}$, as shown below.

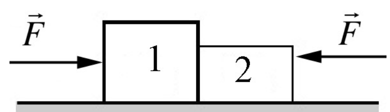


a) Draw a free-body diagram for each of the blocks.

b) What is the acceleration of the blocks?

c) Express, in terms of the quantities given in the figure, the magnitude of the contact force between the two blocks. Briefly explain why your make sense.

Suppose now a force of equal magnitude but opposite direction is applied to the block on the right.



d) What is the acceleration of the blocks?

e) What is the magnitude of the contact force between the two blocks in this case? Briefly explain why your make sense.

Problem 1 Solutions:

a) Take the positive direction to be to the right in the figure below.

There is a contact force $\vec{C}$ of magnitude $C \equiv \left|\vec{C}\right|$ on each block; this force is directed to the left on block 1 and to the right on block 2. There is no friction; the vertical normal forces (between the table and the blocks) cancel the weights, and so vertical forces will be neglected.

b) From Newton's Third Law, the acceleration of the block on the left is given by

$$m_1 a_1 = F - C \tag{1.1}$$

and the acceleration of the block on the right is given by

$$m_2 a_2 = C. \tag{1.2}$$

Note that the sign assigned to C is different for the different blocks. Adding Equations (1.1) and (1.2), and setting $a_1 = a_2 = a$ (the blocks move together) yields

$$\begin{aligned}(m_1 + m_2)a &= F \\ a &= \frac{F}{m_1 + m_2}.\end{aligned} \tag{1.3}$$

It should not be surprising that when the blocks move together, the common acceleration is that of a single object of mass $m_1 + m_2$ subject to a single force of magnitude F.

c) Substituting the result given in (1.3) into either of Equations (1.2) or (1.1) yields, after minor algebra,

$$C = F\frac{m_2}{m_1 + m_2}. \tag{1.4}$$

Note that if $m_2 \ll m_1$, $C/F \to 0$; relatively little force is needed to accelerate the much smaller block. In the limit $m_1 \ll m_2$, $C \to F$; the presence of the small block does not affect the acceleration of the large block.

d) The free body force diagrams onm each block for this case are shown below.

Equations (1.1) and (1.2) become

$$\begin{aligned}m_1 a_1 &= F - C \\ m_2 a_2 &= C - F.\end{aligned} \tag{1.5}$$

Adding these expressions and using $a_1 = a_2 = a$ again yields $a = 0$ immediately; as expected, if there is no net force on the system, there is no acceleration.

e) Using $a_1 = a_2 = a = 0$ in either expression in Equation (1.5) gives $C = F$; neither block accelerates, so there must be zero net force on each block.

Problem 2: Forces Responsible for Acceleration of Car

When a car accelerates forward on a level roadway, which force is responsible for this acceleration? State clearly which body exerts this force, and on which body (or bodies) the force acts.

Problem 2 Solution: The friction force of the road acting on the drive wheels accelerates the car forward. These days, virtually all cars are front-wheel drive, so the road acting on the front wheels causes the acceleration.

Problem 3: Spring Scale

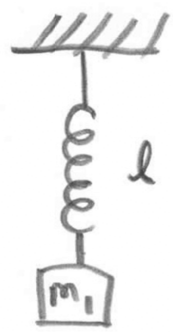
You are standing on a spring bathroom scale in an elevator and you look at the scale while the elevator is at rest with respect to the ground. Describe how the scale readings change as the elevator uniformly accelerates, moves at a constant velocity, and then uniformly decelerates. Explain your answer.

Problem 3 Solution:

The spring scale measures the contact force that the scale exerts on you by converting how much the spring is compressed into a scale reading (often a dial). By Newton's Third Law, the scale applies an equal and opposite force on your feet, which we shall refer to as the contact force on you. The difference between the contact force and the gravitation force must equal the product of your mass and your acceleration; the contact force is greater than the gravitation force and the scale reading increases. When you are moving at constant speed, the contact force is equal to the gravitation force, hence the scale reading returns to its original value. When you decelerate, the contact force is now less than the gravitation force since your acceleration is now in the direction of the gravitation force. Thus the scale reading decreases from the original value.

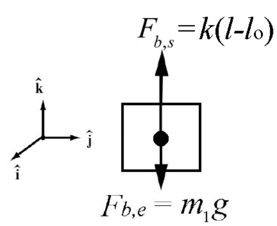
Problem 4: Hooke's Law

Consider a spring with negligible mass that has an unstretched length $l_0 = 8.8\times10^{-2}\,\mathrm{m}$. A body with mass $m_1 = 1.5\times10^{-1}\,\mathrm{kg}$ is suspended from one end of the spring. The other end (the upper end) of the spring is fixed. After a series of oscillations has died down, the new stretched length of the spring is $l = 9.8\times10^{-2}\,\mathrm{m}$. Assume that the spring satisfies Hooke's Law when stretched. What is the spring constant?



Problem 4 Solution:

There are two forces acting on the body: the gravitational force between the body and the earth, $\vec{\mathbf{F}}_{\mathrm{b,e}} = -m_1\, g\, \hat{\mathbf{k}}$, and the force between the body and the spring, $\vec{\mathbf{F}}_{\mathrm{b,s}} = F_{\mathrm{b,s}}\, \hat{\mathbf{k}}$. The interaction between the body and the spring stretches the spring a distance $(l - l_0)$ from its equilibrium length. By Hooke's Law the magnitude of this force is $F_{\mathrm{b,s}} = k(l - l_0)$. The force acting on the spring is thus $\vec{\mathbf{F}}_{\mathrm{b,s}} = k(l - l_0)\,\hat{\mathbf{k}}$. The force diagram for the spring is shown below.



Since the spring is in static equilibrium, the sum of the forces is zero,

$$\vec{\mathbf{F}}_{\mathrm{b,total}} = \vec{\mathbf{F}}_{\mathrm{b,e}} + \vec{\mathbf{F}}_{\mathrm{b,s}} = -m_1\, g\, \hat{\mathbf{k}} + k(l - l_0)\,\hat{\mathbf{k}} = \vec{\mathbf{0}}. \tag{4.1}$$

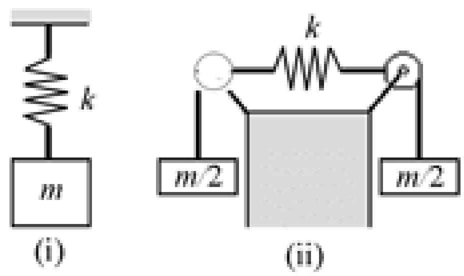
We can solve this equation for the spring constant,

$$k = \frac{m_1 g}{l - l_0} = \frac{(1.5\times10^{-1}\ \text{kg})(9.8\ \text{m}\cdot\text{s}^{-2})}{9.8\times10^{-2}\,\text{m} - 8.8\times10^{-2}\,\text{m}}$$

$$=147\ \text{N}\cdot\text{m}^{-1}. \qquad (4.2)$$

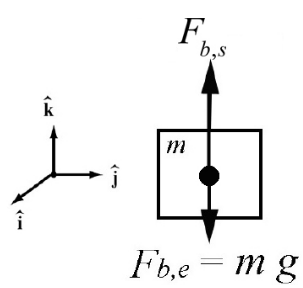
Problem 5: Force Hooke's Law

A body of mass m is suspended from a spring with spring constant k in configuration (a) and the spring is stretched $0.1\,\text{m}$. If two identical bodies of mass $m/2$ are suspended from a spring with the same spring constant k in configuration (b), how much will the spring stretch? Explain your answer.



Problem 5 Solution:

In part (a), assume that the spring is directly connected to the body. There are two forces acting on the body: the gravitational force between the body and the earth, $\vec{\mathbf{F}}_{\text{b.e}} = -mg\,\hat{\mathbf{k}}$, and the force between the body and the spring, $\vec{\mathbf{F}}_{\text{b.s}} = F_{\text{b.s}}\,\hat{\mathbf{k}}$. (The spring is actually connected to the rope, but since the rope is massless, the tension in the rope is uniform and the spring force transmits through the rope so the tension in the rope is equal to the magnitude of the spring force). The interaction between the body and the spring stretches the spring a distance x from its equilibrium length. By Hooke's Law the magnitude of this force is $F_{\text{b,s}} = k\,x$ and so the force acting on the spring is $\vec{\mathbf{F}}_{\text{b,s}} = k\,x\,\hat{\mathbf{k}}$.
The force diagram on the body is shown below



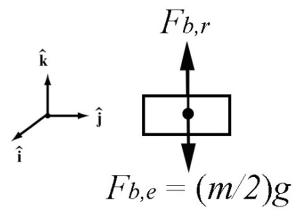
Theses two forces on the body balance since the body is in equilibrium, and so

$$\vec{\mathbf{F}}_{\text{b}}^{T} = \vec{\mathbf{F}}_{\text{b.e}} + \vec{\mathbf{F}}_{\text{b.s}} = -mg\,\hat{\mathbf{k}} + k\,x\,\hat{\mathbf{k}} = \vec{\mathbf{0}} \tag{5.1}$$

Therefore the spring stretches a distance

$$x = mg / k . \qquad (5.2)$$

In configuration (b), the forces acting on the body are the gravitational force between the body and the earth, and the force between the rope and the body. The force diagram is shown in the figure below.



Since the body is in static equilibrium,

$$\vec{\mathbf{F}}_{\mathrm{b}}^{T} = \vec{\mathbf{F}}_{\mathrm{b.e}} + \vec{\mathbf{F}}_{\mathrm{b.r}} = -\left(mg/2\right)\hat{\mathbf{k}} + F_{\mathrm{b,r}}\ \hat{\mathbf{k}} = \vec{\mathbf{0}} . \qquad (5.3)$$

Therefore the magnitudes of the force of the rope (tension in the rope) and the gravitational force on the body are equal;

$$mg / 2 = F_{\mathrm{b,r}} . \qquad (5.4)$$

Suppose the spring stretches by an amount x_1. Just as in part (a), the tension in the rope is uniform, so the tension in the rope is equal to the magnitude of the spring force.

$$F_{\mathrm{b.r}} = k\, x_1 . \qquad (5.5)$$

Combining Equations (5.4) and (5.5) yields

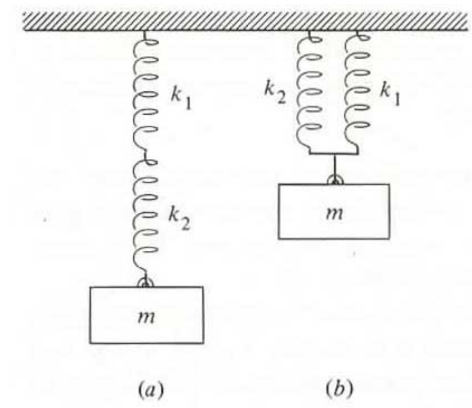
$$k\, x_1 = (m/2) g . \qquad (5.6)$$

Thus the spring stretches by half the amount as in part (a), as given by Equation (5.2),

$$x_1 = (m/2) g / k = x / 2 . \qquad (5.7)$$

Problem 6: Equivalent Spring Constants

Find the effective spring constants for the two systems shown in figures (a) and (b). The block has mass m and the two springs having spring constants k_1 and k_2 respectively,



The spring with spring constant k_1 is attached to the ceiling and one end of the spring with spring constant k_2, and the other end of the second spring is attached to the block (the springs are attached in *series*).

a) Each spring is attached to the ceiling and the block (the springs are attached in *parallel*).

Problem 6 Solution:

a) Each spring is stretched by a (possibly) different extension. Let Δx_1 represent the extension of the upper spring and Δx_2 represent the extension of the lower spring. The total extension for both springs is

$$\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2 . \tag{6.1}$$

The magnitude of the spring force is the same in each spring; that is, the spring force can be considered as being transmitted uniformly through the two springs. The two springs can be considered as a single spring. The spring force is for a massless spring is then

$$\left|\vec{\mathbf{F}}\right| = k_1 \Delta x_1 = k_2 \Delta x_2 \tag{6.2}$$

Note that this last equation implies that $\Delta x_1 = \left|\vec{\mathbf{F}}\right| / k_1$ and $\Delta x_2 = \left|\vec{\mathbf{F}}\right| / k_2$. Since the total extensions add, Equation (6.1) becomes

$$\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2 = \frac{\left|\vec{\mathbf{F}}\right|}{k_1} + \frac{\left|\vec{\mathbf{F}}\right|}{k_2}. \tag{6.3}$$

The equivalent spring is under the same tension (same spring force), $\left|\vec{\mathbf{F}}\right| = k' \Delta x_{\text{total}}$, where k' is the equivalent spring constant when the two springs are replaced by one. The total displacement is therefore

$$\Delta x_{\text{total}} = \frac{\left|\vec{\mathbf{F}}\right|}{k'}. \tag{6.4}$$

Substitute Equation (6.4) for the total displacement into Equation (6.3), yielding

$$\frac{\left|\vec{\mathbf{F}}\right|}{k'} = \frac{\left|\vec{\mathbf{F}}\right|}{k_1} + \frac{\left|\vec{\mathbf{F}}\right|}{k_2}. \tag{6.5}$$

The magnitude $\left|\vec{\mathbf{F}}\right|$ of the common force cancels from Equation (6.5), and the equivalent spring constant k' is given by

$$\frac{1}{k'} = \frac{1}{k_1} + \frac{1}{k_2}. \tag{6.6}$$

The spring constants for springs connected in series add *inversely*.

b) When springs are connected in parallel (side by side), each spring adds to the total force that an equivalent spring would apply,

$$F_{\text{total}} = F_1 + F_2. \tag{6.7}$$

Since both springs stretch the same amount Δx, spring 1 exerts a force $F_1 = k_1 \Delta x$ and spring 2 exerts a force $F_2 = k_2 \Delta x$. Therefore the total force in Equation (6.7) becomes

$$F_{\text{total}} = k_1 \Delta x + k_2 \Delta x = (k_1 + k_2)\Delta x. \tag{6.8}$$

The equivalent spring (by definition) would be held under the same spring force F_{total} and stretched the same amount Δx, so the equivalent spring constant is found from

$$F_{\text{total}} = k' \Delta x. \tag{6.9}$$

Substituting Equation (6.9) into Equation (6.8) yields

$$k' \Delta x = k_1 \Delta x + k_2 \Delta x \,. \tag{6.10}$$

Therefore the equivalent spring constant for two springs connected in *parallel* adds;

$$k' = k_1 + k_2 \,. \tag{6.11}$$

Problem 7: Pulling a Rope Attached to Two Trees

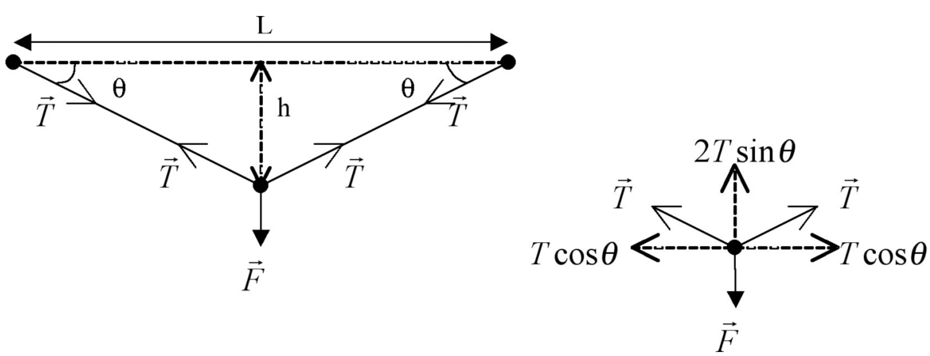
Suppose a rope is tied rather tightly between two trees that are 30 m apart. You grab the middle of the rope and pull on it perpendicular to the line between the trees with as much force as you can. Assume this force is 1000 N (about 225 lb), and the point where you are pulling on the rope is $h = 1\,\text{m}$ from the line joining the trees.

a) What is the magnitude of the force tending to pull the trees together?

b) Give an example of a situation where you think this may be of practical use.

Problem 7 Solutions:

a) The various forces involved are shown in Figure 2. The $\hat{\mathbf{i}}$-direction is to the right in the figure, and the $\hat{\mathbf{j}}$-direction is down in the figure, in the direction of the applied force $\vec{\mathbf{F}}$. Note that the figure is not to scale ($L/h \sim 30$).



The equations for force equilibrium, at the point where the force is applied, are:

$$\begin{aligned} \hat{\mathbf{i}} &: T\cos\theta - T\cos\theta = 0 \\ \hat{\mathbf{j}} &: \quad 2T\sin\theta - F = 0. \end{aligned} \tag{7.1}$$

From the second equation in (7.1), $T = F/(2\sin\theta)$. With, $L = 30\,\text{m}$ and $h = 1\,\text{m}$,

$$\tan\theta = h/(L/2) = 1/15 \Rightarrow \theta = \tan^{-1}(1/15) = 3.81° \Rightarrow \sin\theta = 0.067\,. \tag{7.2}$$

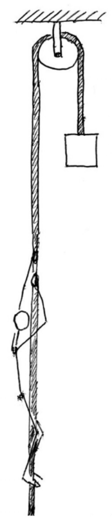
Note that the small angle approximation $\sin\theta \sim \tan\theta = h/(L/2)$ is certainly valid in this situation. The tension in the rope is then given by

$$T = \frac{F}{2\sin\theta} = (1000\,\text{N}) \times 15/2 = 7500\,\text{N}\,. \tag{7.3}$$

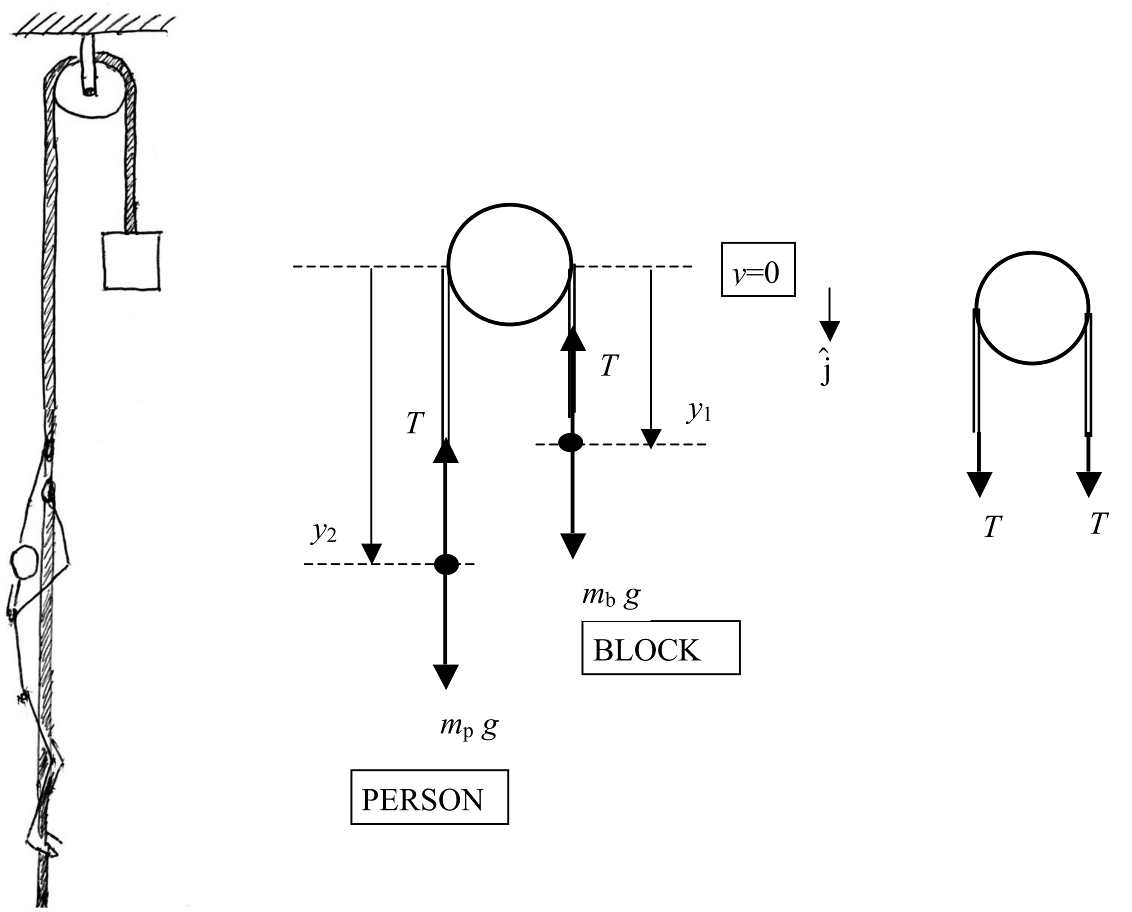
b) If the applied force is vertically downward, you’re walking a tightrope, and the trees should be big enough to hold up to roughly 7-8 times your weight. If the applied force is horizontal, maybe one of the trees will come down, and you might have firewood without having to resort to a chainsaw. If one end of the rope is attached to a large enough tree and the other to a car in a ditch, you can apply several times the pulling force to try to move the car.

Problem 8: Climbing a Rope

A person clings to a rope (assumed massless) that passes over a pulley. The person is balanced by a block of mass m hanging at the other end of the rope. Initially both the person and block are motionless. The person then starts climbing the rope by pulling on it with a constant force in order to reach the block. The person moves a distance L relative to the rope. Does the block move as a result of the person's climbing? If so, in which direction and by how much?



Problem 8 Solution:



The force diagrams are shown in the above figure. As the person pulls up on the rope, there is a force down on the rope, creating a tension in the rope. This tension is transmitted through the rope, and so is also the force on the object. Both the person and block satisfy Newton's Second Law,

$$m\,g - T = m\,a_y\,; \tag{8.1}$$

the person and the block accelerate upwards with the same acceleration.

The length of the rope between the person and the block is

$$l = y_1 + \pi R + y_2\,, \tag{8.2}$$

where R is the radius of the pulley. As the person climbs, y_1 and y_2 change by the same (negative) amount. So, if a length of rope L passes through the person's hands, both the person and the object rise a distance $L/2$.