PROFESSOR: I want to pursue in this segment the case, whereby the resistive force is proportional to the speed and not to the speed squared. So what I would advise you to do is to revisit the segment where I discuss the falling oil drop, very small oil drop, where the resistive force is by far dominated by the linear term in the speed. And the question that I want to ask now is how does the speed change with time?

Earlier we calculated the terminal velocity for the oil drop, but how does the speed change with time is a question that I have not addressed.

So here is my oil drop. It has a certain mass m. And I release it at speed 0, so at t equals 0, my velocity in this direction equals 0. I will call this the plus direction. And I drop in 1 atmosphere air. It is a sphere, and so I can use my equations that we discussed earlier. For a spherical object, the force on that object, F equals ma, is the gravitational force in this direction minus c1r times v in that direction, where the object has a velocity, a speed v in this direction. It experiences in the opposite direction a resistive force, which in this case, specifically chosen, is proportional to the speed.

c1 at 1 atmosphere air is approximately 3.1 times 10 to the minus 4 kilograms per meter per second. And the terminal velocity, which we discussed earlier in a different segment is when a becomes 0, the terminal velocity therefore equals mg divided by c1r.

This terminal velocity is reached after a certain amount of time. If we take an oil drop, which I took before in a different segment with a radius of 10 to the minus 6 meters, which is 1 micron, and the oil drop has a mass of roughly 3.3 times 10 to the minus 15 kilograms. Because the density for oil was about 800 kilograms per cubic meters, then we would find for the terminal velocity approximately 10 to the minus 4 meters per second. It's about 0.1 millimeters per second.

How will it reach this terminal velocity? If I make a plot of this velocity of the oil drop as a function of time-- this is the origin-- then very schematically, it will go like this and it will ultimately reach the terminal velocity.

And I want to discuss now what is the representation of this curve? How does this velocity, how does this speed, change with time? In other words, I have to evaluate the equation ma equals mg minus c1r times v. And I'm going to write down for v, which is dy dt-- I call this the y-direction. I'm going to write down for that y dot. This is my shorthand notation. And I'm going to write down for a, which is the acceleration in the y-direction, which is d2y dt squared. I'm going to write for that the shorthand notation y double dot. So my equation then becomes m y double dot plus c1r y dot minus mg equals 0. And I can divide by m, and then I get a differential equation y double dot plus c1r divided by m times y dot minus g equals 0.

This is a differential equation, which holds the secret to how the velocity changes with time. For a given object with a given radius and a given mass, this is a constant. And if this is air at 1 atmosphere, we know the value for c1.

What is the solution to this differential equation? I will give you the solution. In a separate segment, I discuss solutions to differential equations. Here I will simply give you the solution.

Here's a differential equation. y dot equals the maximum value, which is the terminal velocity times 1 minus e to the power minus t divided by tau. And I will get back to this.

y double dot, the second derivative, then becomes, this first term is a constant so that is 0 the derivative. But the second term, I get v terminal. I get 1 over tau. I get a minus 1 over tau plus another minus sign, so that the minus signs disappear. e to minus t divided by tau. And these two values now I'm going to substitute into this equation. So this one comes here and this one comes here, and I have my minus g.

So when I do that I get v terminal divided by tau times e2 to minus t over tau. That's this first term. Here you see it.

Now comes the second term. Plus c1r divided by m times v term. Remember the y dot has this term and it has this term. So I have to also add minus c1r divided by m v term e to the minus t divided by tau minus g, and that whole thing equals 0.

Now, look at this equation. This is a term, which depends on t and this is a term that depends on t. So this term minus this term must be 0 because this holds at all moments in time. If it holds at all moments in time, this one minus this one must be 0, and this one minus g must also be 0.

Well, if I first take these two, my v term cancels. They have that in common. My e to the minus t over tau cancels. They have that in common. And so I find that 1 over tau minus c1r m equals 0. Therefore, tau equals m divided by c1r.

And if I take this relationship, then I find c1r divided by m times the terminal speed equals g minus g equals 0. So I can write equals g. So v terminal becomes mg divided by c1r.

We already knew that earlier. We have derived that in a separate segment, but we find that here we got it for free. And so the solution now of the velocity-- this is the velocity in the y-direction as a function of time becomes mg divided by c1r times 1 minus e to the power minus t c1r divided by m. Wow! And this is the terminal velocity.

Let us check whether we meet our initial conditions. When we substitute in here t equals 0, then this term is 1. 1 minus 1 is 0. We find that v indeed, is 0. That's a must because we released it at 0 speed. So we must meet that initial condition.

When we wait a long time and we go-- we make t infinitely large, this term becomes 0 and v then becomes the terminal velocity, which is exactly what we would expect. So if w make a plot of the velocity versus time, and this is the value of the terminal velocity, then this curve has this form. The terminal velocity here equals mg divided by c1r. And if we wait tau seconds, and tau equals m divided by c1r, if we wait so many seconds, then the speed that the object has is about 63% of the terminal velocity.

So let's now substitute in there the values that we had for the oil drop and see what the value of tau is. I'd like to remind you that the mass of the oil drop was 3.3 times

10 to the minus 15 kilograms. The value for c1 is 3.1 times 10 to the minus 4 kilograms per meter per second. And the radius of the oil drop was 1 micron. And if I substitute that in here, then I find tau is approximately 1.1 times 10 to the minus 5 seconds. In other words, 11 microseconds. So in 11 microseconds, the speed of the oil drop in 11 microseconds is already 63% of the terminal velocity.

By the way, if I had chosen a larger oil drop, for instance, if the radius had been 3 microns, so 3 times 10 to the minus 6 meters, then the value for tau would have been roughly 10 to the minus 4 seconds. About 1/10 of a milliseconds. Because notice, I have an m upstairs and m of course goes with r cubed. I have an r downstairs, so tau goes with r squared. So if I make r three times larger, then tau would become nine times larger.

Now comes the $64 question, how long will it take now for this oil drop to reach the terminal velocity? Well, that comes down to the evaluation of the time dependent term in the velocity. And this time dependent term equals 1 minus e to the minus t over tau.

If we evaluate this time dependent term alone, then when t equals 0, you get 1 minus 1. And so this term becomes 0. There is 0 speed. Because this, the speed is proportional to this term, remember?

If you put in t equals tau, then you would get 1 minus 1 over e. And that is what we just calculated. That is about 0.63. So you know it's 63% of terminal velocity.

Let's now take 3 tau. Now you get 1 minus 1 divided by e to the power 3. And that is approximately 0.95. So now you are at 95% of the terminal speed.

Let's go to 5 tau. Well, you do your homework and you will find that the value of 1 minus e to the minus t over tau is now 0.993. So you are at 99.3% of the terminal speed.

So how long do you have to wait now to reach the terminal speed? Well, you would have to wait infinitely long. But of course, for all practical purposes, I would say it depends a little bit on your own taste. That after maybe 3 tau or 5 tau you are close enough that you can say, well, we have really reached the terminal velocity.

If we simply make a plot of the term 1 minus e to the minus t over tau. So this value would be 1, this is time and this is 0. Then we have here this function. And then, at 1 tau you are at 0.63 and at 3 tau you are already at a value of 0.95. And so in our case, for the oil drop, we had something like 10 to the minus 5 seconds for tau. Well, if you wait 5 tau seconds, then you would already be 99.3% of the terminal speed. For me, that's close enough. If it's not close enough for you, well then you have to wait a little longer. Now you may ask, all right, we know the velocity as a function of time. Do we know now also the position as a function of time? Yeah, we do. We are within spitting distance of having the position as a function of time. But I would like you to work that out.

The velocity as a function of time dy dt for which we wrote v, for which we have written y dot has the form terminal velocity times 1 minus e to the power minus t over tau. And at t equals 0, we have to be at y equals 0, and the speed is 0.

We know tau as we calculated it. We know what the terminal speed is. We calculated that too. How do we find now y as a function of time? Where it is as a function of time? Well, that becomes an integral and I leave it up to you to execute that integral. It's not the hardest one. This integral from 0 to t 1 minus e to the minus t over tau dt.

And if you can solve this integral, as I said, it's not the hardest one. It has two terms. Then you have the position y as a function of time. So I suggest that you do that. And if you prefer not to, be my guest.