

MITOCW | MIT8_01SCF10mod08_05_300k

As I have discussed in a separate segment, the resistive force that a spherical object experiences when it moves through air at 1 atmosphere can be written as follows. This is the magnitude of the resistive force, $c_1 r v$ plus $c_2 r^2 v^2$. v is the speed of the object and r is the radius of the sphere, the object has the shape of a sphere. And if we deal with 1 atmosphere air, then c_1 is approximately 3.1×10^{-4} kilograms per meter per second. And c_2 is approximately 0.87 kilograms per cubic meter. This has the dimension of density.

Now as I mentioned in a different segment, in most cases that do deal with in our daily life, the v^2 term dominates. However, if you go to very small radii, very small objects, you can be in a domain where the linear term dominates. And that's the case I want to discuss now.

I take a very, very small oil drop and the oil drop has a radius of about 1 micron, which is 10^{-6} meters. And the mass of the oil drop is then approximately 3.3×10^{-15} kilograms. The density of oil is approximately 800 kilograms per cubic meters.

If now I want to know at what critical speed these two terms are identical, then I can call that the critical speed. And the critical speed is given by c_1 divided by c_2 times r . That is the speed at which these two terms are equal. Don't confuse that with the terminal speed. It's a speed whereby this linear term has the same magnitude as this term in v^2 .

If I calculate that for the oil drop, I find that the critical speed is about 360 meters per second. That is a speed, which is extraordinarily high and our oil drop will never even come close to that speed. Therefore, I'm now operating in the domain whereby the linear term is dominating by far. The linear term is the one that counts here.

So if now I want to evaluate the motion of the oil drop, I can ignore the square of the velocity term. I can only accept then the linear term. And so the equation that now governs the motion is that the force F equals ma is now mg minus $c_1 r v$. The minus sign because the resistive force always opposes the velocity. c_1 is a positive value, [INAUDIBLE] and v is a positive value. So the minus sign takes care of the fact that it is really a vector notation whereby the resistive force opposes the velocity.

So here we have this object and there is the force mg down and there is the resistive force in the opposite direction $c_1 r v$. And the net force is the vectorial sum, is given by this equation.

If now you want to calculate the terminal velocity, that's the case when this is 0. When mg is the same as $c_1 r v$. And so the terminal velocity now for the case, whereby the resistive force is linear in speed, not proportional to the speed square, we now get mg divided by $c_1 r$. And if you want to substitute for

the mass $\frac{4}{3} \pi r^3$ times the density of the object, then you can also write down here $\frac{4}{3} \pi r^2$ times g times ρ divided by c^1 .

We have an r^3 upstairs, we have an r downstairs, so we get an r^2 here. And so you can calculate now what the terminal velocity is for this oil drop. And you will find that for our oil drop-- and I would remind you that it had a radius of 1 micron. This is the mass and this is the density. I find a very low value of approximately 10^{-4} meters per second, which is about $1/10$ of a millimeter per second. So it would take 1,000 seconds-- that is about 17 minutes for this oil drop to fall over a distance of about 10 centimeters. So I release it at 0 speed. I let it go and then, very quickly will it reach the terminal velocity and it'll take 17 minutes to fall over this distance. The reason why I chose these oil drop is that Millikan did a very famous experiment in which he calculated or measured, I should say, the charge of the electron. And he did that during the famous Millikan oil drop experiment. And that's why I thought it was appropriate and interesting to calculate the terminal velocities of these small oil drops, which play a very important role in his experiment.

I mentioned that the falling oil drop very quickly reaches its terminal velocity. Now the \$64 question is, how quickly? I will deal with that in a separate segment.