I now want to discuss what happens when I drop an object in air and it starts to fall, and it experiences a resistive force.

Let us assume that the mass of the object is m. For simplicity we will assume it is a sphere. And let us use 1 atmosphere of air when we plug in the numbers. We have all the ingredients available for 1 atmosphere of air.

So here is that object, and the object experiences the gravitational force, which is mg. At a certain moment of time it has a certain speed, v. And as a result of this speed, it experiences a resistive force. And the magnitude of the resistive force is upwards. That resistive force for a sphere equals c1 r times v, which is this speed plus c2 r squared times v squared. It has one term linear in v and one term, which is proportional to v squared.

If I call this the plus direction and this the minus direction, then the net force on this object mass little m, Newton's law, F equals ma. So that force ma equals mg in the positive direction minus the resistive force minus c1 rv minus c2 r squared v squared. So this is my one-dimensional representation. I don't have arrows over here to remind you that they are vectors because the minus signs take care of that. c1 is a positive number, v is a positive number and c2 is a positive number, and then the minus signs automatically take care of the direction.

Now as the speed increases you see that this resistive force increases. And so there comes a time that the net force, ma, becomes 0. And that's the moment that the speed of this object will not further increase and it has reached then what we call the terminal speed or the terminal velocity. So when ma equals 0, v becomes v terminal. And I will write for that abbreviated v term.

So you start off with 0 velocity and the velocity gradually increases until it reaches this terminal velocity. So if I make a sketch rather qualitatively of the velocity as a function of time, so there comes a time that the velocity is the maximum value possible and you get something like this. And again, the shape that I draw here is a little arbitrary.

At time t equals 0 the value for dv dt is the gravitational acceleration g. So that is at t equals 0. But you see as time goes on, the acceleration goes down and down and down. And right here, the acceleration is almost 0. Of course, it takes infinitely long, maybe two weeks. But nevertheless, there comes a time that through a good approximation the acceleration is 0. And then, the speed will no longer increase.

What will the terminal velocity be? Well, that will be when the gravitational force becomes equal to do resistive force. So when mg equals c1 r times the terminal plus c2 r squared v squared terminal. When this is the case, then the net force on the object becomes 0. The acceleration, a, becomes 0.

Now when you solve this equation, you're going to get two roots, a positive and a negative root for the terminal velocity. And the negative root, of course, you will ignore. But it is one equation with one unknown, so you should be able to solve this.

If you have two spheres with exactly the same radius, but they have different masses, then the one with the largest mass will have the highest terminal velocity. You can see that immediately from this equation.

If I do have here these two spheres with the same radius and they have the same speed at this moment, this one has already reached its terminal velocity m2g, and the resistive force exactly pairs this one out. So here we have a situation where my deceleration in the y direction is 0. The sum of all forces on the object is 0, and the velocity is already terminal velocity.

Here for this object, which has a much larger mass, we have a gravitational force m1g. The resistive force is the same because remember, I said that the velocity, the speed in both cases were identical. So now you see that the net force on this object is not 0 at all; there's still a component in this direction. And so, the acceleration in this direction is not 0. And so the speed is still increasing. In this case, it has not reached its terminal speed. And so, the object with the larger mass will end up with a larger terminal speed than the one with the smaller mass.

Now very often the v squared term will dominate. Very often this term will dominate. If you want to know at what critical velocity this term becomes more important than the linear term, then you have to compare the linear term c1 r v with c2 r squared v squared.

If you want to know when this one becomes larger than this one or vice versa, you make them equal to each other, and then you can solve for the velocity, the critical velocity, which is the crossover point. And that critical velocity then becomes c1 divided by c2 r. And if you put in the numbers that I've given you earlier for c1 and c2 for 1 atmosphere of air, you will find 3.6 times 10 to the minus 4 divided by r. So this is 1 atmosphere of air, and we are dealing with spheres.

You can see here that if r is very, very small, that the critical velocity is very high. And therefore, for very small objects, the chances are that the linear term may dominate, and I will do an example of that. But it's an exception. In general, the kind of daily experience that we have it is the v squared term that dominates.

Let us take a pebble and the pebble is a sphere and the radius of that pebble is 1 centimeter. So that is 10 to the minus 2 meters. The mass of that pebble is 10 grams, so that is 10 to the minus 2 kilograms.

If I calculate the critical velocity, which is the speed whereby the linear term in the velocity and the square term in the velocity are equal, are equally important, I find 0.036 meters per second. So that is at a speed of 3.6 centimeters per second.

Now, if I drop that pebble, I release it at 0 speed, and I wait 1/20 of a second, then the pebble has moved-- and you can check that for yourself-- about 1/2 an inch. And it has a speed, which is already 1/2 a meter per second, which in way larger than this critical speed. So it's obvious that the v squared term dominates. In fact, at this very moment if you calculated the resistive force, you will find something of the order of 2 times 10 to the minus 5 newtons, which is way, way smaller than the gravitational force mg on the object, which is 10 to the minus 1 newtons.

So, for all practical purposes, we can just assume that the entire motion, apart from the very, very, very beginning, but the entire motion is dictated by the resistive force, which is proportional to v squared. And so, the acceleration that we will see for the pebble, F equals ma equals mg minus c2 r squared v squared. So I only have here the v squared term.

And what is now the terminal velocity, the terminal speed for this spherical pebble in 1 atmosphere of air? Well that is when this becomes 0. When ma becomes 0. And so that's the case when the terminal velocity becomes the square root of mg divided by c2 r squared.

Now if we do this in air, then c2 is 0.87. And so for our pebble, we can substitute the mass, and we know g. You take 10 meters per second square. We know c2, we know the radius. You will find that this is approximately 34 meters per second, which is about 75 miles per hour. So that's the terminal velocity for the pebble in 1 atmosphere of air. Now notice that for a given radius, the higher the mass of the object is, the larger the terminal velocity. Which I think is very intuitive. And for a given mass, the larger the radius is, the smaller will be the terminal velocity, the terminal speed. And I think that is also intuitive. .

I have here a ball, which is made of lead. Here it is. And it weighs 60 grams. It has a radius of 1 centimeter. I have also here a beach ball, which also weighs 60 grams. But this beach ball has a radius of 14 centimeters. And let's compare now the terminal velocity between this lead ball, which has a radius of 1 centimeter, and this beach ball, which has a radius of 14 centimeters. And the mass of both part are the same.

So we have the lead ball, which has a radius of about 1 centimeter and the mass is 60 grams. And we have the beach ball provided radius is about 14 centimeters and the mass is also about 60 grams.

What is the terminal velocity for the lead sphere? Well, we calculated the terminal velocity for our pebble, and our pebble had a radius of 1 centimeter. So that was the same. And now we have a mass of 60 grams and the pebble had a mass of about 10 grams. So the lead sphere is six times more massive. And the terminal velocity goes with the square root of m. So the terminal velocity for the lead sphere will be the square root of six times larger than 75 miles per hour. And that will be approximately 190 miles per hour.

The beach ball will have a terminal velocity of, very roughly, 14 miles per hour. And the reason being that the object, the lead sphere and the beach ball have the same mass, but the radius of the beach ball is fourteen times larger. And so the terminal velocity is fourteen times smaller. Because it goes as the square root of 1 over r squared, so it goes as 1/r. So it must be fourteen times smaller than this value, and that's about 14 miles per hour.

If you throw this lead sphere from Empire State Building and you throw the beach ball from Empire State Building, well you may enjoy if the beach ball hits you on the head. You know, 14 miles per hour, that's not so bad. You may not like the lead ball to hit you. In fact, if it reached 190 miles per hour, it could kill you. However, before you decide that it reaches the terminal velocity, you have to be a little careful. The Empire State Building is 300 meters high.

So how now do we know that the lead ball has already reached after 300 meters the terminal velocity? Well, it hasn't. If you assume that there is no air at all, then it takes about 7.8 seconds for the lead ball to hit the ground. So we assume there is no air now. We throw it from the Empire State Building, 7.8

seconds later it hits the ground. And the speed with which it would hit the ground is about 170 miles per hour. So, because of the resistance of the air flow, the speed will be a little less than 170 miles per hour and it will take a little longer than 7.8 seconds to reach the ground. And so it hasn't reached 190 miles per hour yet. But I can assure you that the speed will be high, and not too different from 170 miles per hour. So the bottom line is that the terminal velocity of course, will be reached ultimately, but you have to give it enough space. You have to give it enough time. And in the case of the Empire State Building, that would not be the case for the lead ball. But for sure it would be the case for the beach ball.

Terminal velocity in the case that the v squared term dominates, is the square root of mg divided by c2 r squared. We deal with spheres. Now obviously, if I had a sphere with a uniform mass density, I could replace the mass by 4/3 pi r cubed rho. rho is the mass density of the sphere. And so I could rewrite this if that is convenient, 4/3 pi r rho divided by c2. Oops. When I watched this segment, I noticed that when I introduced the value of rho that in my equation of the terminal velocity I dropped my g. I'll put it in right now.

In this equation there's a g. Sorry for that. You see you have an r squared here and you get an r cubed upstairs who you only end up with an r under the square root here. And so the terminal velocity is then proportional to the square root of r and rho. So two balls made of the same material, the same density, the largest of the two has the highest terminal velocity.

If I have two objects with the same radius but with very different mass density, for instance, I have a lead ball and one made of styrofoam, which could have a difference in density which could easily be a factor of 150. Then, the lead ball would have a much higher terminal velocity. Some twelve times higher than the styrofoam ball with the same radius. It goes with the square rot of the density.

Very practical would be perhaps to calculate what the terminal velocity is of a raindrop. We'll assume that the raindrops are spherical, and let us take them with a radius of about 2 millimeters. 2 times 10 to the minus 3 meters. And the density of water is about 1,000 kilograms per cubic meter.

And when you use these numbers, and you take 1 atmosphere of air, so you use a value for c2, which was approximately 0.87, then you'll find that the terminal velocity for raindrops of this size is roughly 5.4 meters per second. So that is about 12 miles per hour. So raindrops hit you typically at a speed of 12 miles per hour. But of course, it does depend on the radius. If the radius is smaller, if they are smaller, the terminal velocity will be lower.

Suppose you jump out of an airplane-- I hope with a parachute. But suppose the parachute fails. What would then roughly be the terminal velocity of a person? It's a nasty thought perhaps, but it's only an experiment that we do in our head.

I mentioned earlier that the resistive force depends on the shape of the object. And a human body is not really a sphere. But let's just assume that it can be approximated by a sphere. So let's assume that the person is a sphere with a radius of roughly 0.4 meters. The density-- I'll take the mean density of a person. We're almost all made of water, so the mean density is approximately the same as that of water. It's 1,000 kilograms per cubic meter. It's a little more because when you go into a swimming pool, you sink. But it's close enough for all practical purposes. And so, now you can calculate the terminal velocity of a person at 1 atmosphere in air. And that is approximately 175 miles per hour. So when you slam into the ground with 175 miles per hour, that obviously is fatal. You need the parachute.

I would like you to calculate what the terminal velocity is of a ping pong ball. I have a ping pong ball here. It's a nicely colored one. A ping pong ball has a radius of about 1.8 centimeters. And I weighed this ping pong ball. It weighs about 2.6 grams. So please calculate for yourself the terminal velocity of a ping pong ball.

The v squared term will certainly dominate in air at 1 atmosphere. And the mass of the ping pong ball is 2.6 grams. And the radius of the ping pong ball is about 1.8 centimeters. And you will find that it's a very modest speed.

Now if you throw a ping pong ball off the Empire State Building, it may not reach the ground with terminal velocity because there may be winds around the Empire State Building. In fact, there may be times that the ping pong ball may even go up. So the calculations hat we have made of course, make the assumption that the air itself is not in motion. I wouldn't count on that by throwing a ping pong ball off the Empire State Building.