If an object moves into a resistive medium, that could be air, some other gas, or it could be a liquid, it experiences a resistive force, which depends on its speed. This is very, very different from friction that we dealt with. The friction, for instance, when an object slides down a slope, that fictional force is independent of speed. But the resistive force that you experience when you move through a medium, it's like a wind. When you move your hand I can feel this resistive force. When you swim in the water you can feel that force. When you ride a motorcycle, you can really fell that force due to a wind that is created by your own motion. That resistive force clearly depends on your speed.

The resistive force depends on the shape of the object. It depends also on the size of the object. It comes perhaps intuitive that if the size is larger you will see that then, the resistive force is larger. It also depends on the density of the medium. That comes also quite intuitive. The higher the density of the medium, clearly the larger the resistive forced will be. And then, as I already stated, it depends on the speed of the object.

Let me write down in general terms the resistive force.

A resistive force when an object moves through a medium-- I will put here res, which stands for resistive-- equals minus k1-- this is the velocity of that object-- minus k2 v squared, and this is the unit vector in the v direction.

What does this mean? It means that if k1 and k2 are positive values, that the resistive force has two terms. One proportional to v and one proportional to v squared. But each of them oppose the direction of the velocity. That's why you see the minus v roof here and the minus v.

Now if we deal with one-dimensional situations, then, in general, we simplify this a little and so we then write that the resistive force equals minus k1 times v minus k2 times v squared. And so the minus signs now clearly indicate the fact that we're dealing with vectors, it indicates that the resistive force opposes the direction of the velocity. k1 and k2 and v in this notation are positive.

In most cases, is this term the one that dominates. There are a few however, where this one dominates. When I have a rain drop falling, this term dominates. I hit a baseball, I throw a pebble, a car going with a speed larger than say, 50 miles per hour, this term dominates. An airplane, no question, this term dominates.

If I drive a car at say, 60 miles per hour, or an airplane flies at a speed of about 600 miles per hour, then the speed is constant. Therefore, the net force on the car and on the airplane is 0. It means that the car must generate a force to overcome the resistive force, so that the net force is 0. And the same is true for the airplane.

If you take a car and you go faster than 50 miles per hour, then the v squared term really dominates. The resistive force is proportional to the density. We will discuss this later in some more detail. And also, the speed squared. It is this term that dominates. And k2 holds, among other things, the density of the medium.

Now if you go from 60 miles per hour down to 55 miles per hour, that is a decrease in the velocity, a decrease in the speed, I should say, of 10%, very roughly. So v squared goes down by roughly 20%. So that means that the resistive force goes down by approximately 20%. So that means that the force that the car, that the engine has to generate to overcome this resistive force is also down by about 20%. Work is force times distance. So you can see that that work that has to be done by the engine is then therefore, 20% less. And so you save 20% fuel. And that was one of the strong arguments that were used several years ago when the speed limit was changed from 60 miles per hour to 55 miles per hour. In practice, the gain also roughly 15% in fuel consumption. But it's still quite substantial.

Compare now 60 miles per hour with 120 miles per hour.

Apart from the fact that it would be your very unwise to go so fast, 120 miles per hour, that's not a very good idea. But apart from that, your speed would be twice as high. That means v squared would be four times as high. And that means chances are that your gas consumption is much higher. The difference is huge. You'd be guzzling up gas.

An airplane that flies at 600 per hour, which is comfortably below the speed of sound, airplanes want to fly high. And the reason why they want to fly high is immediately obvious when you think of the fact that the resistive force is proportional to the density and to the speed squared.

If an airplane is flying at a speed of 600 miles per hour, that's a commercial jet plane. Then at sea level, the density of air is about three times higher than the density of air when you fly at 30,000 feet, which is the normal height for commercial airplanes. A three times lower density at 30,000 feet means the resistive force is three times lower. That means the force that the engines of the plane have to generate is three times lower, and that means the fuel consumption is three times lower. Because work done by the engines is of course, force times distance. And so you see it makes a huge difference for an airplane to fly 600 miles per hour near sea level or to fly 600 miles per hour at an altitude of 30,000 feet. And that's the reason, that's the main reason why these airplanes fly as high as they do.

The values of k that we have seen in the resistive force, they depend on shape of the object, they depend on the size of the object, and they depend on the density rho of the medium.

Now I'm going to restrict myself to spheres. So we have a certain radius, r, and I'm going to restrict myself to air and I do that at 1 atmosphere. k1 is then given by c1 times r and k2 is then c2 times r squared. So my equation becomes that the resistive force equals minus c1 rv. That is the term proportional with the speed v minus c2 r squared v squared. This is the term proportional with v squared.

And when c1 and v and c2 are positive, the minus signs clearly indicate that the resistive force is opposing the direction of the velocity. And in air, at 1 atmosphere, these constants, c1 and c2, have been measured. So we're dealing here with 1 atmosphere of air and I want to remind you that we deal with spheres. c1 is approximately 3.1 times 10 to the minus 4 kilograms per meter per second. And c2 is approximately 0.87 and here the unit is kilograms per cubic meter, which is exactly the same dimension as density. Kilograms per cubic meter has the dimension of density. Now it is not so easy to see where this term comes from. And I will not further discuss this term. This term is much easier to digest. First of all, the r squared term.

If we have a sphere-- this is a sphere. And we move the sphere in this direction, then it experiences a wind.

Well, the cross section of this sphere is obviously proportional to the resistive force. And so you've not surprised that that resistive force is proportional to the surface area, and that goes with r squared. So the r squared term is something that comes in quite natural.

Now how about the v squared term? Imagine that you think of the medium as particles, which have a certain mass m. And they have a certain velocity v, which is your speed. You're moving into that medium and so these particles come to you with a certain speed v. So these particles have a certain momentum. Momentum is mv. And they hit you. And so, since momentum is conserved in the collision, there is momentum transfer. And the momentum transfer depends on how many particles you feel per second on your face and of course, on the mass of these particles.

Now imagine now we take two situations. One whereby the speed is 1 meter per second and the other whereby the speed is twice as high.

Clearly, the momentum of each particle has doubled because of the speed. But the number of particles that hit you every second has also doubled. And so therefore, the momentum transfer per unit time goes up by a factor of 4. So the momentum transfer per unit time, which is a force-- dp dt is a force. Therefore, the momentum transfer per unit time goes as v squared. So we shouldn't be surprised then that we have an r squared term here. That is due to the [? geometry. ?] And that we have a v squared term here. And the c2 holds in it-- buried because you can't quite see that here. But it holds in it the density of the medium.

If we didn't have air, but we had, for instance, water. Then, you can be sure that this c2 would be probably some thousand times larger than this value. Because the density of water is roughly a thousand times higher than the density of air at 1 atmosphere.